p1 Simplified Solar Sail. 1/31/2022

Sun: $L_\odot = 3.8 \times 10^{26}$ J/s

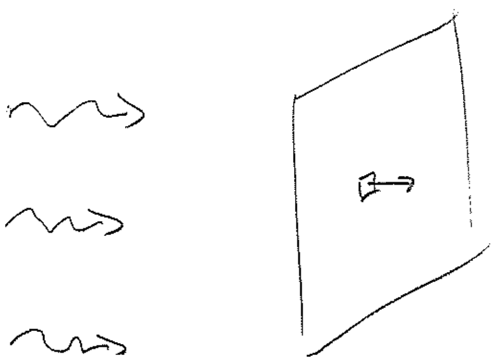
Assume all photons have $\lambda = 5000\,\mathring{A} = 500$ nm.

Energy per photon $E_\nu = \dfrac{1240 \text{ eV}\cdot\text{nm}}{500 \text{ nm}} = 2.48 \text{ eV} * \dfrac{1.609\times10^{-19}\text{ J}}{1 \text{ eV}} = 4\times10^{-19}$ J

Solar flux is $N = \dfrac{L_\odot}{E_\nu} = 9.5\times10^{44} \dfrac{\text{photon}}{\text{s}}$

So at distance R from Sun, flux is

$$f = \frac{N}{4\pi R^2} = \frac{9.5\times10^{44}\ \frac{\text{photon}}{\text{s}}}{4\pi R^2} = \frac{7.58\times10^{43}}{R^2}\ \frac{\text{photon}}{\text{s}\cdot\text{m}^2}$$



Sail of area A is normal to incident solar radiation. Then it intercepts

$$\text{photons intercepted} = \frac{7.58\times10^{43}}{R^2}\ \frac{\text{photon}}{\text{s}\cdot\text{m}^2}\cdot A$$

Each photon is absorbed (no reflection for this sail).

Momentum of 1 photon $p_\nu = \dfrac{E}{c} = \dfrac{hc/\lambda}{c} = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}}{500\times10^{-9}\ \text{m}}$

$= 1.33\times10^{-27}$ kg·m/s / photon

Total momentum absorbed per second

$$= \left(1.33\times10^{-27}\ \frac{\text{kg}\cdot\text{m}/\text{s}}{\text{photon}}\right)\left(\frac{7.58\times10^{43}}{R^2}\ \frac{\text{photon}}{\text{s}\cdot\text{m}^2}\cdot A\right)$$

$$= \frac{1.00\times10^{17}}{R^2}\ \frac{\text{kg}\cdot\text{m/s}}{\text{s}\cdot\text{m}^2}\cdot A$$

p2 But change in momentum per second is equal to force on sail: 1/31/2022

$$\int F(t)\,dt = F \int dt = \Delta p$$

$$\Rightarrow \frac{\Delta p}{\Delta t} = F$$

So force due to 100% absorbed photons is

$$\boxed{F_{abs} = 1.00 \times 10^{17} \frac{A}{R^2} \text{ Newton}}$$

Example: $R = 1\text{ AU} = 1.5 \times 10^{11}\text{ m}$

$A = 1\text{ m} \times 1\text{ m} = 1\text{ m}^2$

$$F_{abs} = 1.00 \times 10^{17} \cdot \frac{1\text{ m}^2}{(1.5 \times 10^{11}\text{ m})^2} = 4.46 \times 10^{-5}\text{ N}$$

If sail made from aluminum foil, of thickness W and density ρ

$\rho = 2700\text{ kg/m}^3$

$W = 100\ \mu\text{m} = 10^{-4}\text{ m}$

Mass of sail $m = A \cdot W \cdot \rho = 1\text{ m}^2 \cdot 10^{-4}\text{ m} \cdot 2700 \frac{\text{kg}}{\text{m}^3} = 0.27\text{ kg}$

Force of grav $F_g = \frac{GM_\odot m}{R^2} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\text{ kg})(0.27\text{ kg})}{(1.5 \times 10^{11}\text{ m})^2}$

$= 0.0016\text{ N} = 1.6 \times 10^{-3}\text{ N}$

In this case, grav > photon pressure

Real sail materials

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IKAROS 7.5 μm polyimide $\sigma \approx 10\ g/m^2 = 10^{-2}\ kg/m^2$

LightSail 2 4.6 μm Mylar

"Salisbury Screen" $\sigma \approx 3\ g/m^2 = 3\times10^{-3}\ kg/m^2$

Note that all these will yield a sail mass so large that grav force > photon pressure force. So none will sail directly away from Sun. However, they can modify a pre-existing orbit, raising or lowering perihelion/aphelion

So, first step is

a) assume some value (realistic) for σ. Say, $\sigma = 5\ g/m^2$

b) assume sail always $\perp$ to radial vector

c) assume sail absorbs all photons

d) assume sail is placed in circular orbit to start

and follow motion of sail over, say, 10 or 100 years.

Computing the components of the force.

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Let's start with coordinates of the Sun and ship

Sun: $(x_\odot, y_\odot, z_\odot)$

ship (x_s, y_s, z_s)

1) compute radial vector $\vec{r}$

$$\vec{r} = (x_s - x_\odot, y_s - y_\odot, z_s - z_\odot)$$

2) compute magnitude of $\vec{r}$

$$|r| = \sqrt{(x_s - x_\odot)^2 + (y_s - y_\odot)^2 + (z_s - z_\odot)^2}$$

3) compute unit vector $\hat{r}$

$$\hat{r} = \frac{\vec{r}}{|r|} = \left(\frac{x_s - x_\odot}{|r|}, \frac{y_s - y_\odot}{|r|}, \frac{z_s - z_\odot}{|r|}\right)$$

Now, Suppose you have calculated the size of the photon force on the sail as F_{photon}. Then the components of force are

$$\vec{F} = F_{photon}\,\hat{r}$$

$$= F_{photon} \cdot \left(\frac{x_s - x_\odot}{|r|}, \frac{y_s - y_\odot}{|r|}, \frac{z_s - z_\odot}{|r|}\right)$$

This calculation is correct only if the sail is face-on to the Sun.