p1

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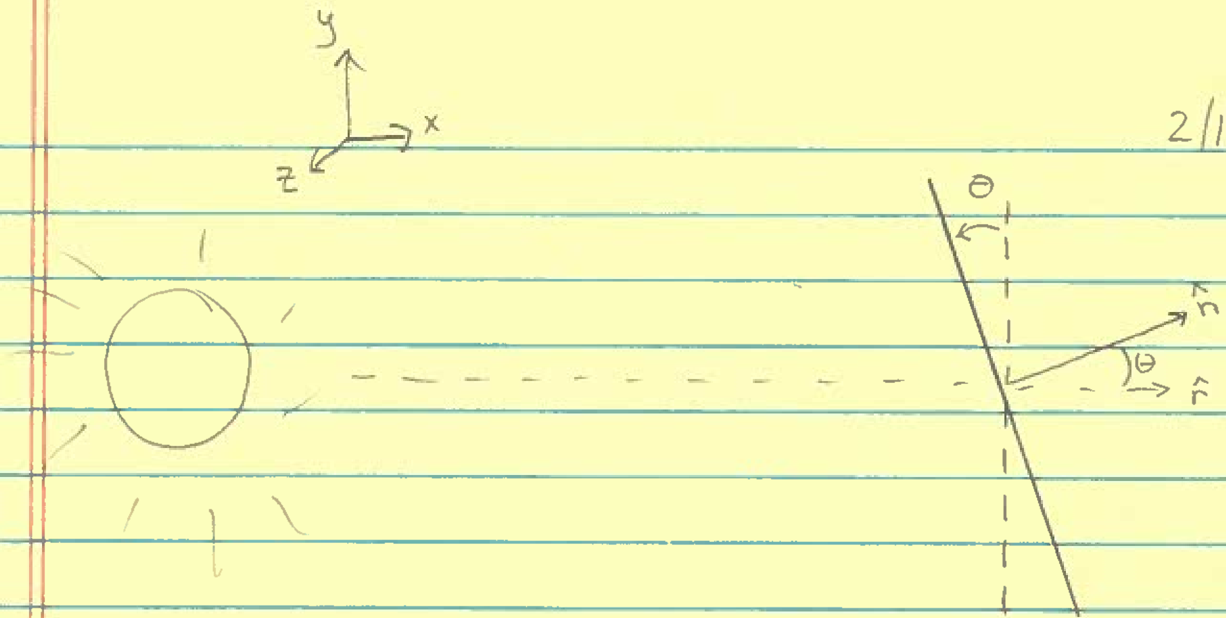

Suppose sail is tilted so it is θ away from face on. Measure θ ccw from radial direction, so normal vector makes angle θ with radial vector.

The collecting area of the sail is now

$$\text{eff collecting area} \quad A_{eff} = A\cos\theta$$

If sail only absorbs photons, this tilt simply decreases the number of photons captured by the sail. There is no change to the direction of momentum transferred to sail.

But if sail also reflects photons, there will also be a change in direction of momentum given to the sail.

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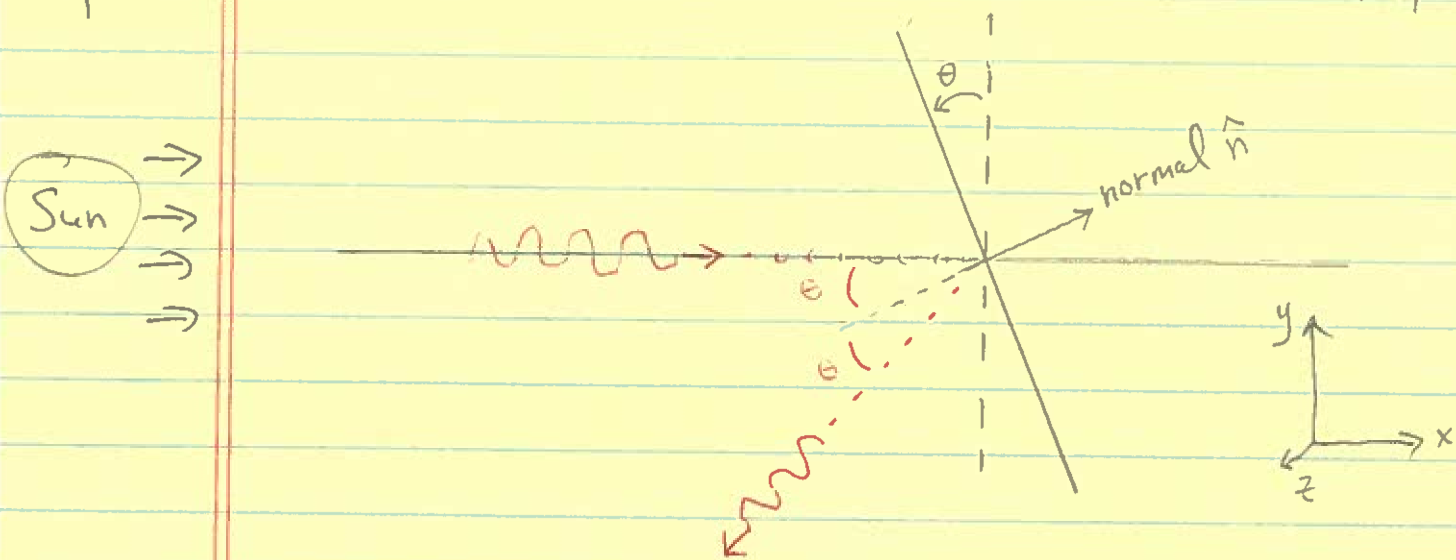


So photon bounces off sail, its return path making an angle of $180° + (180° - 2\theta) = 360° - 2\theta$ with incoming path.

That means

	before x	before y	after x	after y
photon	$+p$	0	$-p\cos(2\theta)$	$-p\sin(2\theta)$
sail	0	0	$+p(1+\cos(2\theta))$	$+p\sin(2\theta)$
total	$+p$	0	$+p$	0

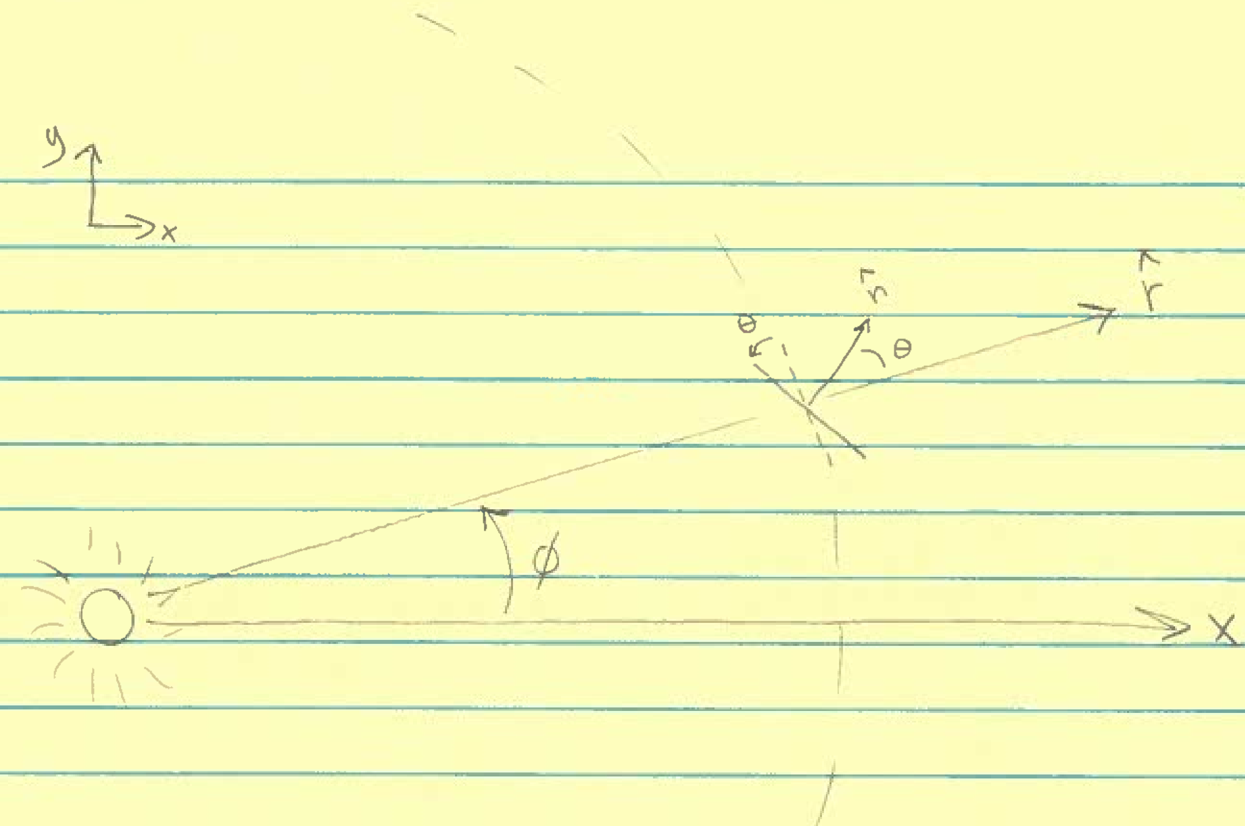
Check: if $\theta = 0$ sail $\Delta p_x = 2p$ $\Delta p_y = 0$ ✓

$\theta = 90°$ $\Delta p_x = 0$ $\Delta p_y = 0$ ✓

The magnitude of the momentum given to the sail is

$$|\Delta p| = \sqrt{p^2(1+\cos(2\theta))^2 + p^2\sin^2(2\theta)}$$

$$= p\sqrt{(1+\cos(2\theta))^2 + \sin^2(2\theta)}$$



If area of sail is $A\ m^2$, we can work out that for 100% absorbant sail, the effective force is

$$\vec{F} = \frac{1.00 \times 10^{17}\ N \cdot A}{R^2}\ \hat{r} \equiv Q \cdot A\ \hat{r}$$

where

$$Q \equiv \frac{1.00 \times 10^{17}\ N}{R^2}$$

So magnitude of effective force when sail is 100% reflective and always is normal to Sun is

$$\vec{F} = 2QA\ \hat{r}$$

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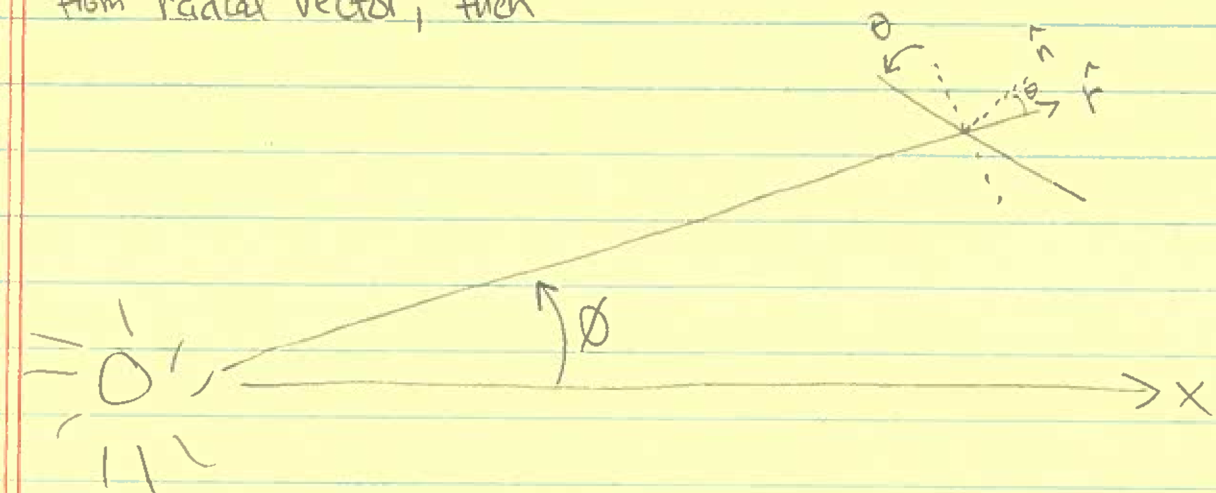
So if a perfectly reflective sail is angled by θ away from face-on, the magnitude must be

$$|F| = 2QA\cos^2\theta$$

and the direction is normal to the sail; that means it is at an angle θ away from radial vector.

If we restrict motion to x-y plane, and choose initial orbit in CCW direction, and define tilt angle θ so it is also CCW away from radial vector, then



radial vector $\hat{r}$ is at angle ϕ from x-axis
normal vector $\hat{n}$ is at angle $(\phi + \theta)$ from x-axis

So direction of force is

$$F_x = |F| \cos(\phi + \theta)$$
$$F_y = |F| \sin(\phi + \theta)$$