Photon vs Energy Magnitude Systems and the Measurement of the Cosmological Parameters

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ABSTRACT

The relative brightnesses of standard candles have long been known to be potentially powerful probes of distance. The distance modulus, the difference between observed and absolute magnitudes, has been associated with the values of the cosmological parameters: Hubble's constant H_0, the mass density Ω_M and the cosmological constant Ω_Λ. In the literature the relationship between these parameters and the distance modulus is calculated for an energy magnitude system; the Johnson-Cousins magnitude system used in observations is in fact a photon-counting system. In this paper, we present the relation between observed and absolute photon magnitudes in terms of the familiar energy distance modulus and derive the correct form of the K-correction. The differences between energy and photon systems are small relative to the measurement errors of contemporary high-redshift supernova searches. The distinction must be made, however, for precision cosmological measurements such as those planned for Type Ia supernovae.

Subject headings: Distance scale — supernovae: general

1. Introduction

Measurements of the cosmological parameters using distance indicators rely on the redshift-dependent evolution of the distance modulus μ. The distance modulus is measured as the difference between observed and absolute magnitudes of a "standard candle" after K-correction (Oke & Sandage 1968) for the redshifting of its spectrum. The theoretical value for μ is related to the luminosity distance $d_L(z)$ defined such that a source with luminosity L at redshift z has observed energy flux f as if the energy has been diluted to the surface of a sphere with radius d_L, i.e. $L = 4\pi d_L^2 f$ (e.g. Carroll, Press, & Turner (1992)). Cosmological parameters can then be measured from their functional dependence on d_L; this technique has been used by two groups with Type Ia supernovae [the High-z team (Riess et al. 1998) and the Supernova Cosmology Project (SCP) (Perlmutter et al. 1999)] and gives evidence for an accelerating universe.

Observations are in fact made with photon counters (CCD's, photo-multipliers) and the luminosity distance is not the same as the "photon luminosity distance" d_γ; if N is the photon luminosity and n is the observed photon flux, then $N = 4\pi d_\gamma^2 n$ where $d_L = d_\gamma(1+z)^{1/2}$. This has lead to some confusion as to whether a "photon" distance modulus should be used to measure cosmological parameters, whether the magnitude system is photon-based or energy-based, and which K-corrections should be applied. Such distinctions which previously have been unimportant are significant as we move into an era of precision cosmology. In this paper we rederive and expand upon the K-correction results of Schneider, Gunn, & Hoessel (1983). We comment on the magnitude system and the Johnson-Cousins system in particular (§ 2). We find that any ambiguity can be removed with the proper definition of the K-correction for which we derive the equations for both photon and energy systems (§ 3). We conclude that although the differences between the two K-corrections are small, the distinction between energy and photon systems is important for planned future high-precision supernova experiments (§ 4).

2. Magnitude Systems

The primary standards of a photometric system can have their magnitudes measured either by their energy or photon flux ratios. Unless a photon–energy conversion correction is later applied, the flux system is determined by the detectors used to measure the primaries. The type of detector used in subsequent observations does not determine whether the magnitude system is photon or energy based; in principle the color and airmass corrections put observed magnitudes into the primary system.

The Johnson-Cousins magnitude system prevalent today is a photon system, what Johnson & Morgan (1953) describe as "a system of photoelectric photometry". As described in Johnson & Morgan (1951), their observational setup employed a photomultiplier as a detector, with the counts being the number of "deflections" recorded by a potentiometer. After an airmass correction these counts were directly converted to magnitudes. The secondary stars of Landolt (1973, 1983, 1992) (whose raw data also were obtained with photon counters) are calibrated via Johnson and Cousins primary standards and thus must be in the photon system. Observed magnitudes are therefore photon-based and should be analyzed as such.

An illustrative example of where there is a numerical difference between the two magnitude systems is a star that has the same integrated B-band energy flux as Vega (which for simplicity we consider to be the zero point of the magnitude system) but has a different photon flux since it has a different spectral energy distribution (SED). Relative magnitude measurements with a single filter of a set of stars with similar spectral energy distributions are independent of whether we are photon counting or measuring energy; two stars with the same SED but differing brightness will have

$$\Delta m = m_2^\gamma - m_1^\gamma = m_2^\epsilon - m_1^\epsilon$$

where m_1 and m_2 are the stars' magnitudes. It follows that since the zeropoint of magnitude system is based on Vega, the energy and photon magnitudes of A0V stars are identical: $m_{A0V}^\gamma = m_{A0V}^\epsilon$.

As an aside, one of the Johnson & Morgan (1953) criteria for a photometric system is "a determination of the zero point of the color indices in terms of a certain kind of star which can be accurately defined spectroscopically." Such knowledge, along with the shapes of the pass-band transmission functions, do allow for calculated transformation between photon and energy magnitude systems. Indeed, much effort has been placed in measuring and modeling the intrinsic SED of Vega (Dreiling & Bell (1980) and references therein).

3. The K-correction

We explicitly review the K-correction calculation of Kim, Goobar, & Perlmutter (1996) that has been used in SCP cosmological analysis. to remove any ambiguity. We define the K-correction K_{xy} such that

$$m_y^\alpha = M_x^\alpha + \mu(z) + K_{xy}^\alpha \tag{1}$$

where $\alpha = \{\gamma, \epsilon\}$ for photon or energy magnitude systems. The observed magnitude in passband y is m_y and the absolute magnitude in passband x is M_x. We adopt the theoretical expression for the distance modulus,

μ, based on luminosity distance. In other words, the functional form of $\mu(z; H_0, \Omega_M, \Omega_\Lambda)$ in Equation 1 is identical for photon and energy systems. Given $f_\lambda(\lambda)$ as the energy flux density of a supernova 10 parsecs away, we can compute the corresponding energy and photon fluxes at high redshift.

$f_\lambda(\lambda)d\lambda$	Energy flux density in dλ bin of a supernova 10 parsecs away
$n_\lambda(\lambda)d\lambda = \frac{\lambda d\lambda}{hc} f_\lambda(\lambda)$	Photon flux in dλ bin of a supernova 10 parsecs away
$f^z_\lambda(\lambda)d\lambda = \frac{d\lambda}{1+z} f_\lambda\left(\frac{\lambda}{1+z}\right)\left(\frac{10pc}{d_L(z)}\right)^2$	Energy flux density in dλ bin of a supernova at z
$n^z_\lambda(\lambda)d\lambda = \frac{\lambda d\lambda}{hc(1+z)} f_\lambda\left(\frac{\lambda}{1+z}\right)\left(\frac{10pc}{d_L(z)}\right)^2$	Photon flux density in dλ bin of a supernova at z

The $(1+z)^{-1}$ terms in the redshifted flux densities are due to wavelength dilution (Oke & Sandage 1968). The ratio between high and low-redshift photon flux is a factor $1+z$ greater than the corresponding ratio for energy flux which suffers from redshifted energy loss. More precisely

$$\frac{n^z_\lambda(\lambda)}{n_\lambda(\lambda/(1+z))} = \frac{(1+z)f^z_\lambda(\lambda)}{f_\lambda(\lambda/(1+z))}. \tag{2}$$

The fact that the relative photon fluxes of high-redshift supernovae are $1+z$ "brighter" than energy fluxes can be interpreted as being due to the latter's extra energy loss due to redshift.

Using the fact that $\mu = -5\log\left(\frac{10pc}{d_L(z)}\right)$ we can compute and compare energy and photon K-corrections,

$$K^\epsilon_{xy} = -2.5\log\left(\frac{\int \mathcal{Z}^\epsilon_x(\lambda)S_x(\lambda)d\lambda}{\int \mathcal{Z}^\epsilon_y(\lambda)S_y(\lambda)d\lambda}\right) + 2.5\log(1+z) + 2.5\log\left(\frac{\int f_\lambda(\lambda)S_x(\lambda)d\lambda}{\int f_\lambda(\lambda/(1+z))S_y(\lambda)d\lambda}\right) \tag{3}$$

$$K^\gamma_{xy} = -2.5\log\left(\frac{\int \lambda\mathcal{Z}^\gamma_x(\lambda)S_x(\lambda)d\lambda}{\int \lambda\mathcal{Z}^\gamma_y(\lambda)S_y(\lambda)d\lambda}\right) + 2.5\log(1+z) + 2.5\log\left(\frac{\int \lambda f_\lambda(\lambda)S_x(\lambda)d\lambda}{\int \lambda f_\lambda(\lambda/(1+z))S_y(\lambda)d\lambda}\right). \tag{4}$$

The filter transmission functions are given as $S_i(\lambda)$ where S_x is the rest-frame filter and S_y is the observer filter. (The transmission functions give the fraction of photons transmitted at a given wavelength where we assume no down-scattering.) For the standard star (i.e. calibrator) SED $\mathcal{Z}(\lambda)$ we assume the existence of a standard star with identical properties as the supernova, i.e. with exactly the same color and observed through the same airmass. Pragmatically, this assumption affirms perfect photometric calibration to all orders of color and airmass. For convenience, we choose these secondary standards to have 0 magnitude. In principle, a different standard will be needed for each filter, choice of photon or energy flux, and each source SED. Each standard is labeled $\mathcal{Z}^\alpha_X$ where $X = \{U, B, V, R, I, \ldots\}$ and $\alpha = \{\gamma, \epsilon\}$ for photon or energy flux as defined earlier.

Equations 3 and 4 generalize the K-corrections of Schneider, Gunn, & Hoessel (1983)[1] and are precisely those given and calculated in Kim, Goobar, & Perlmutter (1996). In that paper, it was found that the differences between the two K-corrections are non-zero but small, $|K^\epsilon_{xy} - K^\gamma_{xy}| < 0.07$ magnitudes. They are a function of redshift, filters, and supernova epoch and thus can cause small systematic shifts in light-curve shapes and magnitude deviations in the Hubble diagram. The use of the incorrect K-correction will have a significant effect on experiments with small $\lesssim 0.1$ targeted magnitude errors.

To illustrate, in Figure 1 we plot $K^\epsilon_{BZ} - K^\gamma_{BZ}$ (where Z refers to the passband and not redshift) for a standard Type Ia supernova at B maximum and 15 rest-frame days after maximum out to $z = 2$. The

[1]Note that in the notation of Schneider, Gunn, & Hoessel (1983), f_ν and $f_{\nu(1+z)}$ are the same function evaluated at different frequencies.

differences are close to zero at $z \sim 1.1$ where $B(\lambda/(1+z)) \sim Z(\lambda)$. Beyond this optimal redshift, the differences can be > 0.01 magnitudes. The redder color of the supernova at the later epoch gives relatively larger photon K-corrections over almost all redshifts.

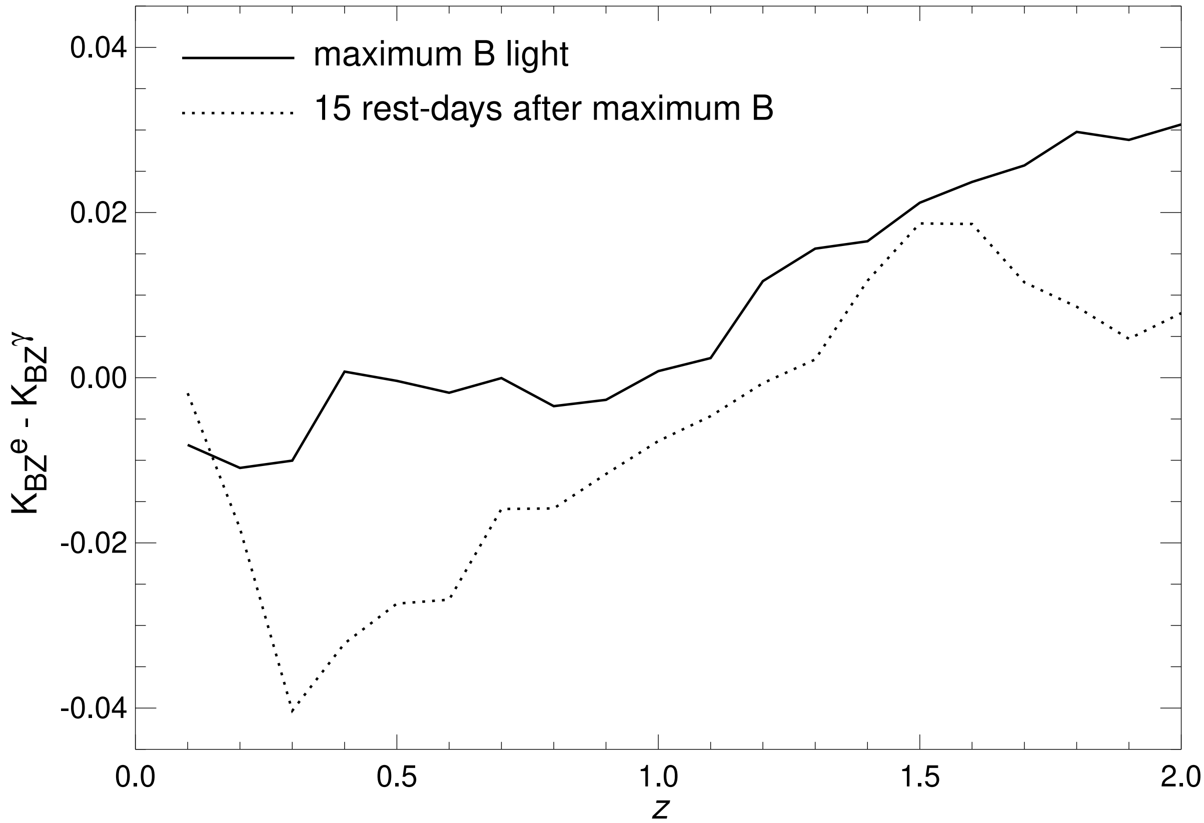

Fig. 1.— $K^{\epsilon}_{BZ} - K^{\gamma}_{BZ}$ for a standard Type Ia supernova at B maximum and 15 days after maximum as a function of redshift. Measurements in I and bluer filters for $z < 1$ supernovae and j and redder filters for $z > 1.5$ would provide a better match of observed spectral regions.

The similarity in the two K-corrections is due to two competing terms that nearly cancel. A photon K-correction is $1 + z$ brighter because the supernova does not suffer redshifting energy loss. However, the zeropoint of the redder filter used to observe the redshifted supernova is larger since an A0V photon spectrum is flatter than its energy spectrum. This makes the observed supernova magnitude numerically fainter. Consider the special case where $S_y(\lambda) = S_x(\lambda/(1+z))$. With perfect filter-matching the specifics of the supernova spectrum are unimportant and the K-corrections depend on the zeropoints:

$$K^{\epsilon}_{xy} = -2.5\log\left(\frac{\int \mathcal{Z}^{\epsilon}_{x}(\lambda)S_x(\lambda)d\lambda}{\int \mathcal{Z}^{\epsilon}_{y}(\lambda)S_y(\lambda)d\lambda}\right) \tag{5}$$

$$K^{\gamma}_{xy} = -2.5\log\left(\frac{(1+z)\int \lambda\mathcal{Z}^{\gamma}_{x}(\lambda)S_x(\lambda)d\lambda}{\int \lambda\mathcal{Z}^{\gamma}_{y}(\lambda)S_y(\lambda)d\lambda}\right) \tag{6}$$

$$= \quad -2.5 \log \left(\frac{(1+z) < \lambda_x > \int \mathcal{Z}_x^\gamma(\lambda) S_x(\lambda) d\lambda}{< \lambda_y > \int \mathcal{Z}_y^\gamma(\lambda) S_y(\lambda) d\lambda} \right) \tag{7}$$

where $< \lambda >$ is the effective wavelength of the standard through the filter. As long as the standard star is well behaved, we expect the effective wavelength of the redshifted filter to be $1+z$ greater then that of the restframe filter $< \lambda_y > \sim (1+z) < \lambda_x >$ so that

$$K_{xy}^\gamma \sim -2.5 \log \left(\frac{\int \mathcal{Z}_x^\epsilon(\lambda) S_x(\lambda) d\lambda}{\int \mathcal{Z}_y^\epsilon(\lambda) S_y(\lambda) d\lambda} \right) = K_{xy}^\epsilon. \tag{8}$$

Choosing filters that accept the same spectral region at both low and high redshifts not only reduce errors but also reduces the difference between energy and photon K-corrections.

The effect of using the "energy" distance modulus in defining the K-correction in Equations 1, 3, and 4 are seen in the open-filter K-corrections. When $S_x = S_y = 1$, the energy K-correction is unnecessary and indeed $K_{xy}^\epsilon = 0$. For the photon K-correction we find $K_{xy}^\gamma = -2.5 \log (1+z)$, the difference between "energy" and "photon" distance moduli.

A simple measure for the difference between single-filter K-corrections is the ratio in effective wavelength of a redshifted and unredshifted source through that filter. For example, sources with power-law SED's have identical photon and energy K-corrections. For low-redshift objects the difference in effective wavelength should be very small (unless they have pathological spectra) and thus make little difference in distance determinations. For example, a Type Ia supernova at maximum at $z = 0.1$ observed through the B-band would have a distance modulus error of 0.02 magnitudes if the wrong K-correction were applied.

4. Conclusion

We have shown that the measurements $m_Y(z) - M_X$ do depend on whether the magnitude system is based on energy or photon flux. Although the "photon luminosity distance" is shorter than the standard luminosity distance, we can still use the relation $m_Y(z) = M_X + \mu(z) + K_{XY}$ with the appropriate definitions of the K-corrections; the ones of Kim, Goobar, & Perlmutter (1996) are appropriate. With this definition, the standard equations linking the energy distance modulus to cosmology are applicable. The Johnson-Cousins magnitude system is in fact photon-based. Therefore, the K_{XY}^γ K-correction should and has been used in the supernova cosmology analysis of the Supernova Cosmology Project. Although application of the incorrect K-correction would contribute negligibly to the error budget of the current supernova sample, the distinction is important for precision experiments that require 0.02 magnitude accuracies, such as the Supernova Acceleration Probe. With the choice of well-matched filters, differences between energy and photon K-corrections can be minimal.

Using the "count" distance modulus based on d_γ in Equation 1 would provide a more physically satisfying definition of the count K-correction. Recall that $\mu^\epsilon = \mu^\gamma + 2.5 \log (1+z)$. Then the extra $2.5 \log (1+z)$ in the K-correction would give

$$K_{xy}^\gamma = -2.5 \log \left(\frac{\int \lambda \mathcal{Z}_x^\gamma(\lambda) S_x(\lambda) d\lambda}{\int \lambda \mathcal{Z}_y^\gamma(\lambda) S_y(\lambda) d\lambda} \right) + 2.5 \log \left(\frac{\int \lambda f_\lambda(\lambda) S_x(\lambda) d\lambda}{\int \lambda' f_\lambda(\lambda') S_y((1+z)\lambda') d\lambda'} \right). \tag{9}$$

In other words, the K-correction would depend simply on the ratio of supernova photons in the rest-frame filter and a blue-shifted observer filter, and the zeropoint. This methodology would preserve the physical meanings that we associate with both distance modulus and K-correction. For simplicity, however, we here adopt the energy distance modulus for both K-corrections to be consistent with the literature and to ensure unambiguity when referring to K-corrected magnitudes and distance moduli.

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