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a) I am so sick of watching the viking's 8b room be absolutely abysmal every week that I trap them in a spherical shell, shrink them down to a radius of 0.05 um and blast them into space. Eventually, they find themselves in a cloud of similarly sized carbon-based dust and atomic H gas in an ordinary part of the ISM, with a stellar energy density of $1.7 \times 10^{-13}\,J/m^3$. What is the temperature of dust in this cloud?

$a = 0.05\,um$, $\varepsilon = 1.7 \times 10^{-13}\,J/m^3$, $\sigma = 5.67 \times 10^{-8}\,W/m^2 \cdot K^4$

$E_{abs} = Q_{abs}\, \varepsilon c\, (\pi a^2)$; $Q_{abs} = 0.8 \left(\frac{a}{0.1\,um}\right)^{0.85}$

$E_{rad} = Q_{emit}\, (4\pi a^2)\, \sigma T^4$; $Q_{emit} = 3.2 \times 10^{-4} \left(\frac{a}{0.1\,um}\right)\left(\frac{T}{20\,K}\right)^2$

$$0.8 \left(\frac{a}{0.1\,um}\right)^{0.85} \varepsilon c (\pi a^2) = (3.2 \times 10^{-4})\left(\frac{a}{0.1\,um}\right)\left(\frac{T}{20K}\right)^2 (\sigma T^4)(4\pi a^2)$$

$$\frac{0.8\, \varepsilon c\, (20K)^2}{(3.2 \times 10^{-4})(4)(\sigma)\left(\frac{a}{0.1\,um}\right)^{0.15}} = T^4$$

$$T = \left(\frac{(0.8)(1.7 \times 10^{-13}\,J/m^3)(3.0 \times 10^8\,m/s)(400\,K^2)}{(3.2 \times 10^{-4})(4)(5.67 \times 10^{-8}\,W/m^2 \cdot K^4)\left(\frac{0.05\,um}{0.1\,um}\right)^{0.15}}\right)^{(1/4)} \approx \boxed{25\,K}$$

b) If the H atoms in the cloud are ~50 K warmer than dust grains and at a density of 35 atoms/cm^3, how long until the 8b room dust particle actually does something useful and collides with 2 H atoms (potentially forming H_2)?

$T_H = 75\,K$, $n_H = 35 \times 10^4$ atoms/m^3, $m_H = 1.67 \times 10^{-27}$, $k = 1.38 \times 10^{-23}\,m^2\,kg/s^2 K$

$$v_H = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{(3)(1.38 \times 10^{-23}\,m^2 kg/s^2 K)(75K)}{(1.67 \times 10^{-27}\,kg)}} = \boxed{1364\,m/s}$$

$$mfp_{dust} = \frac{1}{n_H \pi d^2} = \frac{1}{n_H \pi (2a)^2} = \frac{1}{(35 \times 10^4\,atom/m^3)(\pi)(1 \times 10^{-7}\,m)^2} = \boxed{9.1 \times 10^5\,m}$$

$$\text{time between collisions} = \frac{mfp}{v_H} = \frac{(9.1 \times 10^5\,m)}{(1344\,m/s)} = 667\,s \Rightarrow \boxed{11\text{ minutes}}$$

~11 minutes between collisions ×2 ≈ $\boxed{22\text{ min}}$ to collide with 2 H atoms

↳ 22 min is also roughly equal to 0.5 interceptions for the 8b room! So they could make 2 H_2 molecules in the average time it takes them to throw an int! :)

↳ (every 45 min or so)