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Final Class - Example Problem:

- Determine if a (recently exploded) star will escape the MW given:

↳  $R_{\text{star}} = 8.3 \text{ kpc}$

↳ ~~Galactic longitude~~  $= 180^\circ$

↳ Fe XXVI spectral peak @  $\lambda = 0.1804 \text{ nm}$   
↳ Ly $\alpha$

Part 1: Determine  $v_{\text{esc}}$  for star:

from 1/2:  $\frac{1}{2} M v_{\text{esc}}^2 - \frac{G M_{\text{enc}} M}{R} = 0$ , solved:  $v_{\text{esc}} = \sqrt{\frac{2 G M_{\text{enc}}}{R}}$

need  $M_{\text{enc}}$ :

$$R = 8.3 \text{ kpc} = (8.3 \text{ kpc}) \left( \frac{1000 \text{ pc}}{1 \text{ kpc}} \right) \left( \frac{3.08 \times 10^{16} \text{ m}}{1 \text{ pc}} \right) = 2.566 \times 10^{20} \text{ m}$$

$$v^2 = \frac{M_{\text{enc}} G}{R}, \text{ so } M_{\text{enc}} = \frac{v^2 R}{G} = \frac{(235000 \text{ m/s})^2 (2.56 \times 10^{20} \text{ m})}{6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}} = 2.12 \times 10^{41} \text{ kg}$$

↳ for an ordinary star in a circular orbit, I assume  $v \approx 235 \text{ km/s}$

$$v_{\text{esc}} = \sqrt{\frac{2 (6.67 \times 10^{-11} \frac{\text{N m}^2}{\text{kg}^2}) (2.12 \times 10^{41} \text{ kg})}{(2.56 \times 10^{20} \text{ m})}} = 332.4 \text{ km/s}$$

Part 2: Determine  $v_{\text{star}}$

$\lambda_0$  for Fe XXVI, Ly $\alpha$ :  $\Delta E = z^2 (-13.6 \text{ eV}) \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ ,  $z_{\text{Fe}} = 26$

$$\Delta E_0 = (26^2) (-13.6 \text{ eV}) \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 6.895 \text{ keV}$$

$$\lambda_0 = 1240 \text{ eV nm} / (6.895 \times 10^3 \text{ eV}) = 0.1799 \text{ nm}$$

Doppler Shift:  $\Delta v = c \left( \frac{\lambda - \lambda_0}{\lambda_0} \right)$

$$\Delta v_{\text{star}} = (3 \times 10^8 \text{ m/s}) \left( \frac{0.1804 \text{ nm} - 0.1799 \text{ nm}}{0.1799 \text{ nm}} \right)$$

$$\Delta v_{\text{star}} = 1,001 \text{ km/s}$$

1,001 km/s > 332.4 km/s, escapes!