$$P = n\,k\,T$$

$$\kappa \propto \frac{\rho}{T^{3.5}}$$

$$\lambda = 4L$$

$$f = \frac{v_s}{\lambda} = \frac{v_s}{4L}$$

$$\text{period} = \frac{1}{f} = 4\frac{L}{v_s}$$

$$\text{period} \sim 4\left(\frac{R}{v_s}\right)$$

$$v_s = \sqrt{\frac{T}{\mu}}$$

$$v_s \sim \sqrt{\frac{P}{\rho}}$$

$$P \sim \frac{GM\rho}{R}$$

$$v_s \sim \sqrt{\frac{GM}{R}}$$

$$\text{period} \sim \frac{4R}{\sqrt{\frac{GM}{R}}} = 4\left(\frac{GM}{R^3}\right)^{-1/2}$$

$$L = \left(4\pi R^2\right)\,\sigma T^4$$

$$L \propto R^2 \quad \rightarrow \quad R \propto L^{1/2}$$

$$\text{period} \propto M^{-1/2}\,R^{3/2}$$

$$\text{period} \propto M^{-1/2}\,L^{3/4}$$

$$L \sim M^4 \quad \rightarrow \quad M \sim L^{1/4}$$

$$\text{period} \propto (L^{1/4})^{-1/2}\,L^{3/4} \sim L^{5/8}$$