

e) To find the group velocity as the waves travel together through the oil, we start with the sum of the waves:

$$\text{Sum} = A \sin(k_1'x - \omega_1 t) + A \sin(k_2'x - \omega_2 t)$$

Use trig ID

$$\sin(P) + \sin(Q) = 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$$

So

$$\text{Sum} = 2A \sin\left(\frac{(k_1' + k_2')x - (\omega_1 + \omega_2)t}{2}\right) \cos\left(\frac{k_1' - k_2'}{2}x - \frac{\omega_1 - \omega_2}{2}t\right)$$

↑
use for V_{ph}

↑
use for V_{gr}

$$V_{gr} = \frac{\omega_1 - \omega_2}{k_1' - k_2'} = 2.66 \times 10^8 \frac{m}{s}$$

f) The phase velocity is

$$V_{ph} = \frac{\omega_1 + \omega_2}{k_1' + k_2'} = 2.81 \times 10^8 \frac{m}{s}$$

So $V_{gr} > V_{ph}$ in this instance, and as usual.