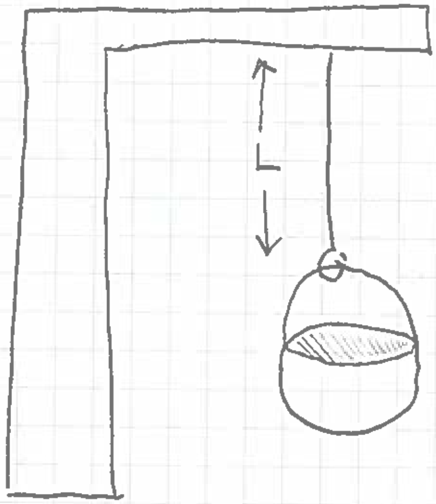


Fred hangs a cauldron of mass $M = 10$ kg from a wire

$L = 2$ m

$m = 0.02$ kg

Fred pulls the wire a distance $d = 5$ mm to the side at middle of wire. When he releases it, it oscillates in fundamental mode.

a) what is freq of this sound?

Tension in wire is approximately (because $M \gg m$)

$$T = Mg$$

So speed of waves in wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{(10\,kg)(9.8\,\frac{m}{s^2})}{(0.02\,kg)/(2m)}}$$

$$= 99 \text{ m/s}$$

We can calc freq via

$$f\lambda = v \rightarrow f = \frac{v}{\lambda} = \frac{99\,\frac{m}{s}}{4m}$$ ← 2L in fundamental

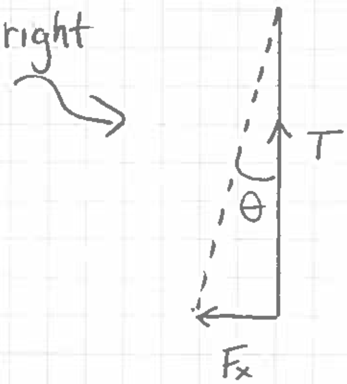
$$\boxed{f = 24.75\text{ Hz}}$$

b) How hard must Fred pull the wire to make this sound?

→

$$\tan\theta = \frac{d}{L/2} = \frac{0.005\ \text{m}}{1\ \text{m}} = 0.005$$

But we can draw a similar triangle showing forces: see figure at right



$$\tan\theta = \frac{F_x}{T}$$

So

$$F_x = T\tan\theta = 98\ \text{N}\,(0.005) = 0.49\ \text{N}$$

Leopold demands that Fred increase frequency to

$$f_{new} = 1.5 f = 1.5\,(24.75\ \text{Hz}) = 37.1\ \text{Hz}$$

c) using sand, Fred puts sand into cauldron, increasing its mass.

Want

$$f_{new} = \frac{v}{\lambda} \rightarrow v = f_{new}\cdot\lambda$$

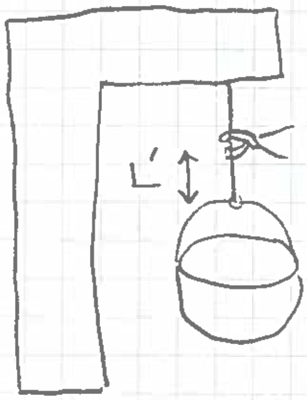
$$\rightarrow \sqrt{\frac{T}{\mu}} = f_{new}\cdot\lambda \rightarrow T = \mu\cdot f_{new}^2\,\lambda^2 = M_{new}\,g$$

$$\rightarrow M_{new} = \frac{\mu f_{new}^2\,\lambda^2}{g} = \frac{\left(0.01\frac{\text{kg}}{\text{g}}\right)\left(37.1\frac{1}{\text{s}}\right)^2(4\text{m})^2}{9.8\ \text{m}/\text{s}^2}$$

$$= 22.5\ \text{kg}$$

So Fred must add 12.5 kg of sand to the cauldron.

d) using pliers, Fred clamps the wire at a point between the cauldron and support.



The new wire has length $L' < L$, but same tension. In order to produce sound with $f_{new} = 37.1\ Hz$, the length L' must be

$$f_{new} = \frac{v}{\lambda} = \frac{v}{2L'}$$ ← new fundamental mode

So

$$L' = \frac{v}{2 f_{new}}$$

$$L' = \frac{99\ m/s}{2(37.1\ Hz)} = 1.33\ m$$