Earthquake! A section of highway ripples as waves flow down it (see figure), at a speed of $v = 50\ m/s$. Assume the measurements are the sum of two sinusoidal waves, with equal amplitudes and speeds.

$$y_1(x,t) = A \sin(k_1 x - \omega_1 t)$$
$$y_2(x,t) = A \sin(k_2 x - \omega_2 t)$$

In our case

$$V_1 = \frac{\omega_1}{k_1} = V_2 = \frac{\omega_2}{k_2}$$

The sum is

$$y_1 + y_2 = A \sin(k_1 x - \omega_1 t) + A \sin(k_2 x - \omega_2 t)$$

Use trig ID

$$\sin(P) + \sin(Q) = 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right)$$

$$\rightarrow \quad y_1 + y_2 = 2A \sin\left(\frac{k_1 x + k_2 x - \omega_1 t - \omega_2 t}{2}\right) \cos\left(\frac{k_1 x - k_2 x + \omega_1 t - \omega_2 t}{2}\right)$$

$$= 2A \sin\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right) \cos\left(\frac{k_1 - k_2}{2} x - \frac{\omega_1 - \omega_2}{2} t\right)$$

$$= 2A \sin\left(k_{avg} x - \omega_{avg} t\right) \cos\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right)$$

high-freq oscillations "inner wiggles"

low-freq oscillations "envelope"

From the graph, period of inner wiggles is

$$P_{inner} \cong 5.5\ \text{sec} \rightarrow \omega_{avg} = \frac{2\pi}{P_{inner}} \cong 1.1\ \frac{rad}{s}$$

Period of envelope is

$$P_{env} \cong 60\ \text{sec} \Rightarrow \Delta\omega = \frac{2 \cdot 2\pi}{P_{env}} \cong 0.21\ \frac{rad}{s}$$

Now we can solve for ω_1 and ω_2

① $\dfrac{\omega_1 + \omega_2}{2} = 1.1 \ \frac{\text{rad}}{\text{s}}$ ② $\omega_1 - \omega_2 = 0.21 \ \frac{\text{rad}}{\text{s}}$

From ② $\omega_1 = \omega_2 + 0.21 \ \frac{\text{rad}}{\text{s}}$

$$\Rightarrow \left(\omega_2 + 0.21 \ \tfrac{\text{rad}}{\text{s}}\right) + \omega_2 = 2\left(1.1 \ \tfrac{\text{rad}}{\text{s}}\right)$$

$$2\omega_2 + 0.21 \ \tfrac{\text{rad}}{\text{s}} = 2.2 \ \tfrac{\text{rad}}{\text{s}}$$

$$2\omega_2 = 2.0 \ \tfrac{\text{rad}}{\text{s}}$$

$$\rightarrow \omega_2 = 1.0 \ \tfrac{\text{rad}}{\text{s}}$$

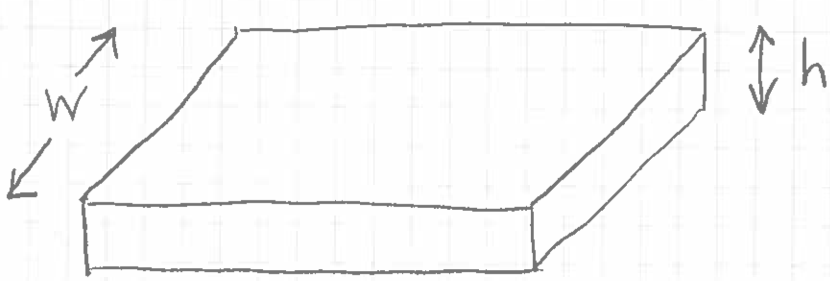
$$\rightarrow \omega_1 = \omega_2 + 0.21 \ \tfrac{\text{rad}}{\text{s}} = 1.2 \ \tfrac{\text{rad}}{\text{s}}$$

Since we know speed $v = 50 \ \frac{\text{m}}{\text{s}}$, we can compute wavelengths:

$$v = \frac{\omega_1}{k_1} \rightarrow k_1 = \frac{\omega_1}{v} \rightarrow \lambda_1 = \frac{2\pi}{k_1} = \frac{2\pi v}{\omega_1}$$

$$\lambda_1 = \frac{2\pi\,(50 \ \text{m/s})}{1.2 \ \frac{\text{rad}}{\text{s}}} = 262 \ \text{m}$$

$$\lambda_2 = \frac{2\pi\,(50 \ \text{m/s})}{1.0 \ \text{rad/s}} = 314 \ \text{m}$$



Road surface is $W = 20$ m wide, and $d = 0.5$ m thick. Average density is
$$\rho = 4000 \ \frac{\text{kg}}{\text{m}^3}.$$

The linear mass density of the road is

$$\mu = hw\rho = (0.5\,m)(20\,m)\left(4000\,\frac{kg}{m^3}\right) = 4\times10^4\,\frac{kg}{m}$$

And the power carried by the wave in this medium is

$$P = \frac{1}{2}\mu v \omega^2 A^2$$

We can compute power seperately for each wave, then add. The amplitude of the sum is

$$2A = 2\,m \Rightarrow A = 1\,m$$

so

$$P_1 = \frac{1}{2}\mu v \omega_1^2 A^2 = \frac{1}{2}\left(4\times10^4\,\frac{kg}{m}\right)\left(50\,\frac{m}{s}\right)\left(1.2\,\frac{rad}{s}\right)^2(1\,m)^2$$

$$= 1.44\times10^6\ W$$

$$P_2 = \frac{1}{2}\mu v \omega_2^2 A^2 = \frac{1}{2}\left(4\times10^4\,\frac{kg}{m}\right)\left(50\,\frac{m}{s}\right)\left(1.0\,\frac{rad}{s}\right)^2(1\,m)^2$$

$$= 1.00\times10^6\ W$$

The total power is thus

$$\boxed{P_{tot} = 2.44\times10^6\ \text{Watts}}$$