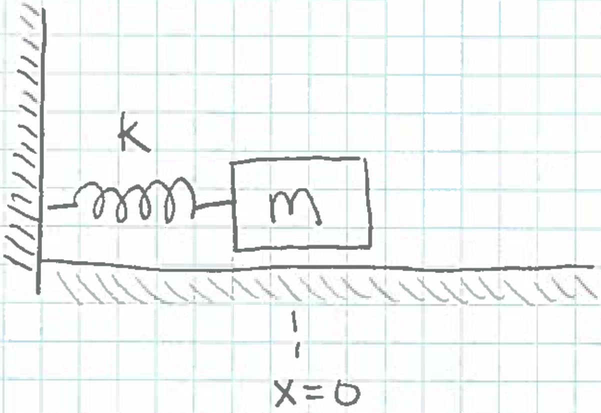


Block of mass m sits on frictionless floor in vacuum chamber, attached to spring of force constant k. Equilibrium at $x=0$.

Suppose block is displaced slightly by distance x from equilibrium, and released.

a) Net force on block is

$$\vec{F}_x = -k\vec{x}$$

b) Differential equation for motion is

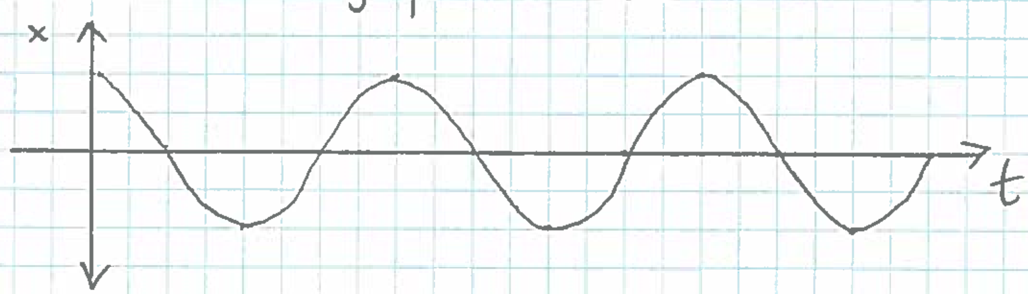
$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x}$$

or

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$$

if we drop vector signs in 1-D problem

c) Position vs. time graph looks like



Now, air is pumped into chamber. Block encounters air resistance

$$\vec{F}_{air} = -b\vec{v}$$

Once again, block is displaced from equilibrium by x and released.

d) $\vec{F}_{net} = -k\vec{x} - b\vec{v}$

e) $$\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{b}{m}\frac{dx}{dt}$$

or

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

f) what value of b will cause block to come to rest in minimum time? The critical damping requires

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \rightarrow 0$$

$$\rightarrow \frac{k}{m} = \frac{b^2}{4m^2} \rightarrow \boxed{b = \sqrt{2mk}}$$

g) Position vs. time in critical damping looks like