Dust particle of mass $m = 2\times10^{-9}$ kg has electric potential energy

$$U(x) = E\left(10 - 5e^{-(x/a)^2}\right)$$

where

$$E = 6\times10^{-11}\ \text{J}$$
$$a = 3.5\times10^{-4}\ \text{m}$$
x = distance from tip of a wire

a) is $x=0$ a stable equilibrium?

$$F_x = -\frac{\partial U}{\partial x} = -\frac{2E}{a^2}\cdot x\cdot 5e^{-(x/a)^2}$$

if $x>0$, F_x is negative, pushes back to 0
$x<0$, F_x is positive, pushes back to 0

yes, $x=0$ is stable equilibrium

b) $$F_x = ma_x \rightarrow a_x = \frac{\partial^2 x}{\partial t^2} = -\frac{2E}{ma^2}\cdot x\cdot 5e^{-(x/a)^2}$$

We want to show

$$\frac{\partial^2 x}{\partial t^2} = -\overset{\text{const}}{(\ \ )}\,x$$

as condition for SHM

But we see $x\cdot e^{-(x/a)^2}$

Fortunately, if x is small, $x \ll a$, we can use an approximation. In general,

$$\text{if } z \ll 1,\ e^{z} \simeq 1 + z + \tfrac{1}{2}z^2 + \dots$$

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