

Dust particle of mass  $m = 2 \times 10^{-9}$  kg has electric potential energy

$$U(x) = E (10 - 5e^{-(x/a)^2})$$

where

$$E = 6 \times 10^{-11} \text{ J}$$

$$a = 3.5 \times 10^{-4} \text{ m}$$

$x$  = distance from tip of a wire

a) is  $x=0$  a stable equilibrium?

$$F_x = -\frac{\partial U}{\partial x} = -\frac{2E}{a^2} \cdot x \cdot 5e^{-(x/a)^2}$$

if  $x > 0$ ,  $F_x$  is negative, pushes back to 0

$x < 0$ ,  $F_x$  is positive, pushes back to 0

Yes,  $x=0$  is stable equilibrium

b)

$$F_x = ma_x \rightarrow a_x = \frac{\partial^2 x}{\partial t^2} = -\frac{2E}{ma^2} \cdot x \cdot 5e^{-(x/a)^2}$$

We want to show

$$\frac{\partial^2 x}{\partial t^2} = -\left(\overset{\text{const}}{\quad}\right) x \quad \text{as condition for SHM}$$

But we see  $x \cdot e^{-(x/a)^2}$

Fortunately, if  $x$  is small,  $x \ll a$ , we can use an approximation. In general,

$$\text{if } z \ll 1, e^z \approx 1 + z + \frac{1}{2}z^2 + \dots$$



So, since  $\frac{x}{a} \ll 1$ ,

$$\begin{aligned}e^{-\left(\frac{x}{a}\right)^2} &\approx 1 - \left(\frac{x}{a}\right)^2 + \frac{1}{2}\left(\frac{x}{a}\right)^4 - \dots \\ &\approx 1 - \frac{x^2}{a^2} \\ &\approx 1\end{aligned}$$

In our case, that means the accel on particle in X-dir is approximately

$$\begin{aligned}\frac{d^2x}{dt^2} &\approx -\left(\frac{10E}{ma^2}\right) x \left[1 - \frac{x^2}{a^2} + \frac{1}{2}\frac{x^4}{a^4} - \dots\right] \\ &\approx -\left(\frac{10E}{ma^2}\right) x [1]\end{aligned}$$

$$\frac{d^2x}{dt^2} \approx -\left(\frac{10E}{ma^2}\right) x \quad \text{therefore SHM}$$

c) Frequency of motion given by

$$\frac{d^2x}{dt^2} \approx -\omega^2 x$$

$$\rightarrow \omega = \sqrt{\frac{10E}{ma^2}} = \sqrt{\frac{10(6 \times 10^{-11} \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2})}{(2 \times 10^{-9} \text{ kg})(3.5 \times 10^{-4} \text{ m})^2}}$$

$$\omega = 1.56 \times 10^3 \frac{\text{rad}}{\text{s}}$$