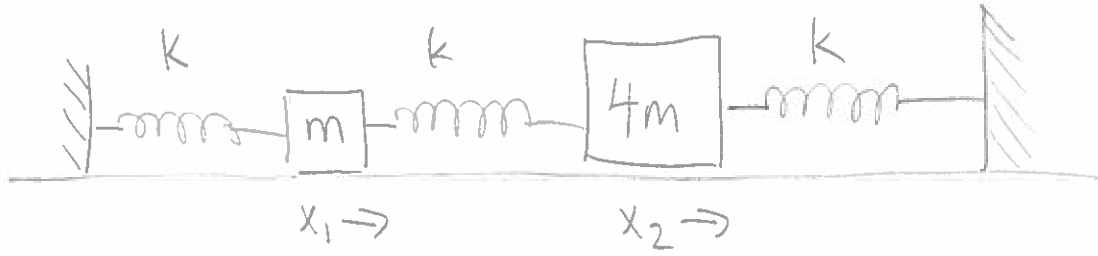

Mass $m = 2kg$, $4m = 8\,kg$ attached to springs of $k = 24\,N$.

$$F_1 = -kx_1 + k(x_2 - x_1) = -2kx_1 + kx_2 = m\frac{d^2x_1}{dt^2}$$

$$F_2 = -k(x_2 - x_1) - kx_2 = +kx_1 - 2kx_2 = 4m\frac{d^2x_2}{dt^2}$$

$$\frac{k}{m}\left[-2 \quad +1\right] x_1 = \frac{d^2x_1}{dt^2}$$

$$\frac{k}{4m}\left[+1 \quad -2\right] x_2 = \frac{d^2x_2}{dt^2}$$

Or, putting it into convenient form

$$\left(\frac{k}{4m}\right)\begin{bmatrix} -8 & +4 \\ +1 & -2 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

If we guess that normal modes of oscillation exist, so that some combination of x_1 and x_2 exhibit SHM, then

$$\frac{k}{4m}\begin{bmatrix} -8 & +4 \\ +1 & -2 \end{bmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2\begin{pmatrix} a \\ b \end{pmatrix}$$

or

$$\begin{bmatrix} -8 & +4 \\ +1 & -2 \end{bmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2\frac{4m}{k}\begin{pmatrix} a \\ b \end{pmatrix}$$

p2. We can define

$$\lambda = -\omega^2 \frac{4m}{k}$$

and then

$$\begin{bmatrix} -8 & +4 \\ +1 & -2 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} - \lambda \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

or

$$\begin{bmatrix} -8-\lambda & +4 \\ +1 & -2-\lambda \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0$$

To find solution, take determinant of this matrix.

$$(-8-\lambda)(-2-\lambda) - (1)(4) = 0$$

$$16 + 8\lambda + 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 + 10\lambda + 12 = 0$$

$$\lambda = \frac{-10 \pm \sqrt{100 - 4(12)(1)}}{2} = -5 \pm \sqrt{13}$$

Now, in terms of ω^2,

$$-\omega^2 \frac{4m}{k} = -5 \pm \sqrt{13}$$

$$\rightarrow \omega^2 = \left[5 \pm \sqrt{13}\right] \frac{k}{4m} = \begin{cases} 1.394 \; k/4m \\ 8.606 \; k/4m \end{cases}$$

Plugging in values for k, and m,

$$\boxed{\omega = \begin{cases} 2.045 \text{ rad/s} = \omega_1 \\ 5.081 \text{ rad/s} = \omega_2 \end{cases}}$$

P3

To find eigenvectors, we plug each frequency into the bottom row of the matrix eqn.

ω_1: $1a - 2b = -\left(\omega_1^2 \cdot \frac{4m}{K}\right) b$

$1a - 2b = -1.394 b$

$\rightarrow a = 0.606 b \Rightarrow s_1 = 0.606 x_1 + 1 x_2$

ω_2: $1a - 2b = -\left(\omega_2^2 \cdot \frac{4m}{K}\right) b$

$1a - 2b = -8.606 b$

$\rightarrow a = -6.606 b \Rightarrow s_2 = -6.606 x_1 + x_2$

$$s_1 = \begin{pmatrix} 0.606 \\ 1 \end{pmatrix} \qquad s_2 = \begin{pmatrix} -6.606 \\ 1 \end{pmatrix}$$

Now, the normal modes can be written as

$$s_1(t) = A_1 \cos(\omega_1 t + \phi_1)$$
$$s_2(t) = A_2 \cos(\omega_2 t + \phi_2)$$

Given initial conditions at $t = 0$

$x_1 = 2\text{ m}$
$x_2 = 0\text{ m}$
$v_1 = 0\text{ m/s}$
$v_2 = 0\text{ m/s}$

we can solve for A_1, ϕ_1, A_2, ϕ_2.

$$s_1(t=0) = A_1 \cos\phi_1 = 0.606 \cdot 2m + 1.0m$$
$$= 1.212 \text{ m}$$
$$ds_1/dt\,(t=0) = -\omega_1 A_1 \sin\phi_1 = 0 \text{ m/s} + 0 \text{ m/s}$$
$$= 0 \text{ m/s}$$

These $\nearrow$ imply

$$A_1 = 1.212 \text{ m}$$
$$\phi_1 = 0 \text{ rad}$$

$$s_2(t=0) = A_2 \cos\phi_2 = -6.606\,(2m) + 1/0m)$$
$$= -13.212 \text{ m}$$
$$ds_2/dt\,(t=0) = -\omega_2 A_2 \sin\phi_2 = 0 \text{ m/s} + 0 \text{ m/s}$$
$$= 0 \text{ m/s}$$

These $\nearrow$ imply

$$A_2 = -13.212 \text{ m}$$
$$\phi_2 = 0 \text{ rad}$$

So

$$s_1(t) = 1.212 \text{ m} \cos(\omega_1 t)$$
$$s_2(t) = -13.212 \text{ m} \cos(\omega_2 t)$$

But

$$s_1 = 0.606\,x_1 + x_2$$
$$-\quad s_2 = -6.606\,x_1 + x_2$$

$$s_1 - s_2 = 7.212\,x_1 + 0\,x_2 \Rightarrow x_1 = \frac{1}{7.212}(s_1 - s_2)$$

Thus

$$x_1(t) = \frac{1}{7.212}\left[1.212\text{m}\cos\left(2.045\,\tfrac{\text{rad}}{\text{s}}\cdot t\right) + 13.212\text{ m}\cos\left(5.081\,\tfrac{\text{rad}}{\text{s}}\cdot t\right)\right]$$