$$\text{dKE} \quad = \quad \frac{1}{2}\, dm\, v^2 \quad = \quad \frac{1}{2}\ (\mu\ dx)\ (v_y^2)$$

$$y(x,t)$$

$$v_y \quad = \quad \frac{\partial y}{\partial t}$$

$$\text{dKE} \quad = \quad \frac{1}{2}\ (\mu\ dx)\ \left(\frac{\partial y}{\partial t}\right)^2$$

$$\text{original length} \quad = \quad dx$$

$$\text{stretched length} \quad = \quad ds \quad = \quad \sqrt{dx^2 + dy^2}$$

$$ds\ -\ dx \quad = \quad \sqrt{dx^2 + dy^2}\ -\ dx$$

$$\begin{aligned} ds\ -\ dx \quad &= \quad \sqrt{dx^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\ -\ dx \\ &= \quad dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ -\ dx \\ &= \quad dx\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}\ -\ dx \\ &\simeq \quad dx\left[1\ + \frac{1}{2}\ \left(\frac{dy}{dx}\right)^2\right]\ -\ dx \\ &\simeq \quad \frac{1}{2}\ \left(\frac{dy}{dx}\right)^2\ dx \end{aligned}$$

$$\left(\frac{dy}{dx}\right)^2\ dx$$

$$\frac{1}{2}\ \left(\frac{dy}{dx}\right)^2\ dx$$

$$\frac{1}{2}\ \left(\frac{dy}{dx}\right)\ dx$$

$$\left(\frac{dy}{dx}\right)\ dx$$

$$ds - dx \simeq \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$\text{Work} = \int \vec{F}\cdot\vec{dx}$$

$$\text{dW} = \vec{F}\cdot\vec{dx}$$

$$\text{dW} = T\,(ds - dx) = T\,\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$\text{dPE} = \frac{1}{2}\,T\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$\text{dE} = \text{dKE} + \text{dPE} = \frac{1}{2}\,(\mu\ dx)\left(\frac{\partial y}{\partial t}\right)^2 + \frac{1}{2}\,T\left(\frac{\partial y}{\partial x}\right)^2 dx$$

$$\frac{\text{dE}}{\text{dx}} = \frac{1}{2}\,\mu\left(\frac{\partial y}{\partial t}\right)^2 + \frac{1}{2}\,T\left(\frac{\partial y}{\partial x}\right)^2$$

$$\Delta x = v\,\Delta t$$

$$\text{(energy of section)} = \frac{dE}{dx}\,\Delta x = \left(\frac{1}{2}\,\mu\omega^2A^2\right)(v\,\Delta t)$$

$$\text{Power} = \frac{\text{energy of section}}{\Delta t} = \frac{\left(\frac{1}{2}\,\mu\omega^2A^2\right)\ (v\,\Delta t)}{\Delta t}$$

$$y(x,t) = A\sin(kx-\omega t)$$

$$\frac{\partial y}{\partial t} = -\omega A\cos(kx-\omega t)$$

$$\frac{\partial y}{\partial x} = kA\cos(kx-\omega t)$$

$$\frac{dE}{dx} = \frac{1}{2}\,\mu\omega^2A^2\cos^2(kx-\omega t) + \frac{1}{2}\,Tk^2A^2\cos^2(kx-\omega t)$$

$$\text{Power} = \frac{1}{2}\,\mu v\omega^2A^2$$

$$\text{Power} = \frac{1}{2}\,(\mu v)\ \left(\omega^2A^2\right)$$

$$\mathrm{KE} \quad = \quad \frac{1}{2}\, mv^2$$

$$\mathrm{PE} \quad = \quad \frac{1}{2}\, k(\Delta x)^2 \quad = \quad \int_0^{\Delta x} k\vec{x} \cdot \vec{dx} \quad = \quad \int_0^{\Delta x} \vec{F} \cdot \vec{dx}$$