$$F_y = -T\sin(\theta) + T\sin(\theta+\Delta\theta)$$

$$F_y = -T(\theta) + T(\theta+\Delta\theta) = T\,\Delta\theta$$

$$\mu = \frac{M}{L}$$

$$F_y = (\mu\,\Delta x)\,\frac{\partial^2 y}{\partial t^2} = T\,\Delta\theta$$

$$\theta = \text{slope of line} = \frac{\partial y}{\partial x}$$

$$\Delta\theta = \frac{\partial\theta}{\partial x}\,\Delta x$$

$$\frac{\partial\theta}{\partial x} = \frac{\partial^2 y}{\partial x^2}$$

$$(\mu\,\Delta x)\,\frac{\partial^2 y}{\partial t^2} = T\,\frac{\partial^2 y}{\partial x^2}\,\Delta x$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\,\frac{\partial^2 y}{\partial t^2}$$

$$\text{speed of disturbance along string}\quad v = \sqrt{\frac{T}{\mu}}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\,\frac{\partial^2 y}{\partial t^2}$$

$$y(x,t) = f(x)\,\cos(\omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 f}{\partial x^2}\,\cos(\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2\,f(x)\,\cos(\omega t)$$

$$\frac{\partial^2 f}{\partial x^2}\,\cos(\omega t) = \left(\frac{1}{v^2}\right) - \omega^2\,f(x)\,\cos(\omega t)$$

$$\frac{\partial^2 f}{\partial x^2} = -\left(\frac{\omega^2}{v^2}\right)\,f(x)$$

$$f(x) = A\cos(\frac{\omega}{v}x) + B\sin(\frac{\omega}{v}x)$$

$$\sin(\frac{\omega}{v}L) = 0 \qquad \Rightarrow \qquad \frac{\omega}{v}L = n\pi$$

$$\frac{\omega}{v} = \frac{n\pi}{L}$$

$$f(x) = B\sin\left(\frac{n\pi x}{L}\right)$$

$$\omega_n = \frac{n\pi v}{L} = \frac{n\pi}{L}\sqrt{\frac{T}{\mu}}$$

$$\omega_1 = \frac{\pi}{L}\sqrt{\frac{T}{\mu}}$$

$$\omega_n = n\,\omega_1$$

$$y(x,t) = B\sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t)$$

$$\text{slope} = \frac{\partial y}{\partial x} = \theta \qquad \Rightarrow \qquad \frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial^2 y}{\partial x^2}$$