$$y(x,t) \quad = \quad f(x,t)$$

$$\sqrt{L^2 + (y_{p+1} - y_p)^2} \quad \sim \quad L$$

$$\sin\left(\frac{y_{p+1} - y_p}{L}\right) \quad \sim \quad \frac{y_{p+1} - y_p}{L}$$

$$\sin(\theta_{p+1}) \quad \sim \quad \frac{y_{p+1} - y_p}{L}$$

$$F_y \quad = \quad T\left(\frac{y_{p+1} - y_p}{L}\right) - T\left(\frac{y_p - y_{p-1}}{L}\right) \quad = \quad \frac{T}{L}(y_{p+1} - 2y_p + y_{p-1})$$

$$\frac{d^2 y_p}{dt^2} \quad = \quad \frac{T}{mL}(y_{p+1} - 2y_p + y_{p-1})$$

$$y_p(t) \quad = \quad A_p \cos(\omega t)$$

$$-\omega^2 A_p \cos(\omega t) \quad = \quad \frac{T}{mL}\left[A_{p+1}\cos(\omega t) \; - \; 2A_p \cos(\omega t) \; + \; A_{p-1}\cos(\omega t)\right]$$

$$-\omega^2 A_p \quad = \quad \frac{T}{mL}\left[A_{p+1} \; - \; 2A_p \; + \; A_{p-1}\right]$$

$$\left(\frac{2T}{mL} - \omega^2\right) A_p \quad = \quad \frac{T}{mL}\left[A_{p+1} \; + \; A_{p-1}\right]$$

$$\frac{A_{p+1} + A_{p-1}}{A_p} \quad = \quad \left[2 \; - \; \frac{mL}{T}\omega^2\right]$$

$$A_p \quad = \quad C\,\sin(p\alpha)$$

$$A_1 \quad = \quad C\,\sin(\alpha)$$

$$A_2 \quad = \quad C\,\sin(2\alpha)$$

$$A_3 \quad = \quad C\,\sin(3\alpha)$$

$$\frac{A_{p+1} \ + \ A_{p-1}}{A_p} \quad = \quad \frac{C\sin\left[(p-1)\alpha\right] \ + \ C\sin\left[(p+1)\alpha\right]}{C\sin\left(p\alpha\right)}$$

$$\sin\beta \ + \ \sin\gamma \quad = \quad 2\sin\left[\frac{1}{2}(\beta+\gamma)\right]\cos\left[\frac{1}{2}(\beta-\gamma)\right]$$

$$C\sin\left[(p-1)\alpha\right] \ + \ C\sin\left[(p+1)\alpha\right] \quad = \quad 2C\sin(p\alpha)\cos(\alpha)$$

$$\frac{A_{p+1} \ + \ A_{p-1}}{A_p} \quad = \quad \frac{2C\sin(p\alpha)\cos(\alpha)}{C\sin\left(p\alpha\right)} \quad = \quad 2\cos(\alpha)$$

$$A_0 \quad = \quad 0 \qquad \Rightarrow \quad C\sin(0\alpha) \quad = \quad 0$$

$$A_{N+1} \quad = \quad 0 \qquad \Rightarrow \quad C\sin\Big((N+1)\alpha\Big) \quad = \quad 0$$

$$(N+1)\alpha \quad = \quad \pi,\ 2\pi,\ 3\pi,\ ...,\ n\pi$$

$$\alpha \quad = \quad \frac{n\pi}{(N+1)}$$

$$A_p \quad = \quad C \ \sin\left(\frac{pn\pi}{N+1}\right)$$

$$y_p(t) \quad = \quad C \ \sin\left(\frac{pn\pi}{N+1}\right) \ \cos(\omega t)$$

$$2\cos\left[\frac{n\pi}{N+1}\right] \quad = \quad \left(2-\frac{mL}{T}\omega^2\right)$$

$$\left(\frac{mL}{T}\right) \ \omega^2 \quad = \quad 2 \ - \ 2\cos\left[\frac{n\pi}{N+1}\right]$$

$$\omega^2 \quad = \quad \left(\frac{2T}{mL}\right)\left[1 \ - \ \cos\left(\frac{n\pi}{N+1}\right)\right]$$

$$2\sin^2(\beta) \quad = \quad 1 \ - \ \cos(2\beta)$$

$$\beta \quad = \quad \frac{n\pi}{2(N+1)}$$

$$\omega_n^2 \quad = \quad \left(\frac{2T}{mL}\right) 2\sin^2\left(\frac{n\pi}{2(N+1)}\right)$$

$$
\begin{aligned}
\omega_n &= \sqrt{\frac{4T}{mL}}\,\sin\left(\frac{n\pi}{2(N+1)}\right) \\
v &= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{M/L}}
\end{aligned}
$$