

If the end of the string is free to slide along a rod, then the boundary conditions change: now

1: y position of incident wave = y pos of reflected wave

2: y velocity " " " = y vel " " "

We know incident wave is

$$y_i(x,t) = A \sin(kx - \omega t)$$

and reflected wave must be some combination of

$$y_r(x,t) = C \sin(kx + \omega t) + E \cos(kx - \omega t)$$

So, at boundary, where $x=0$

$$y_i(0,t) = y_r(0,t)$$

$$A \sin(-\omega t) = C \sin(\omega t) + E \cos(\omega t)$$

$$-A \sin(\omega t) = C \sin(\omega t) + E \cos(\omega t)$$

Once again, we can separate the sine and cosine terms

$$-A \sin(\omega t) = C \sin(\omega t) \Rightarrow \boxed{A = -C}$$

$$E \cos(\omega t) = 0 \Rightarrow \boxed{E = 0}$$

Note that in this case, the amplitude of the reflected wave is opposite in sign to incident wave.