There is an incoming wave

$$y_i(x,t) = A\sin(kx - \omega t)$$

When it hits a wall, it can create reflections, which could be

$$y_2(x,t) = C\sin(kx + \omega t)$$

$$y_4(x,t) = E\cos(kx + \omega t)$$

or any mixture of these ↗. What are the proper values of coeffs C and E? Use the boundary at $x=0$, where the rope is tied to the wall $\Rightarrow$ $y(x=0) = 0$

Sum is $\quad y_i(x,t) + y_2(x,t) + y_4(x,t) = 0$

$$A\sin(kx - \omega t) + C\sin(kx + \omega t) + E\cos(kx + \omega t) = 0$$

Plug in $x=0$

$$A\sin(-\omega t) + C\sin(+\omega t) + E\cos(+\omega t) = 0$$

Here, note a $\sin(\omega t)$ term cannot cancel a $\cos(\omega t)$ term at all values of time t, so we must have separately

sin terms : $\quad A\sin(-\omega t) + C\sin(\omega t) = 0$

cos terms : $\quad E\cos(\omega t) = 0$

$$\Rightarrow \boxed{E = 0}$$

$$-A\sin(\omega t) + C\sin(\omega t) = 0$$

$$\Rightarrow \boxed{A = C}$$

If the end of the string is free to slide along a rod, then the boundary conditions change: now

1: y position of incident wave = y pos of reflected wave

2: y velocity " " " = y vel " " "

We know incident wave is

$$y_i(x,t) = A\sin(kx-\omega t)$$

and reflected wave must be some combination of

$$y_r(x,t) = C\sin(kx+\omega t) + E\cos(kx-\omega t)$$

So, at boundary, where $x=0$

$$y_i(0,t) = y_r(0,t)$$

$$A\sin(-\omega t) = C\sin(\omega t) + E\cos(\omega t)$$

$$-A\sin(\omega t) = C\sin(\omega t) + E\cos(\omega t)$$

Once again, we can separate the sine and cosine terms

$$-A\sin(\omega t) = C\sin(\omega t) \Rightarrow \boxed{A=-C}$$

$$E\cos(\omega t) = 0 \Rightarrow \boxed{E=0}$$

Note that in this case, the amplitude of the reflected wave is <u>opposite</u> in sign to incident wave.