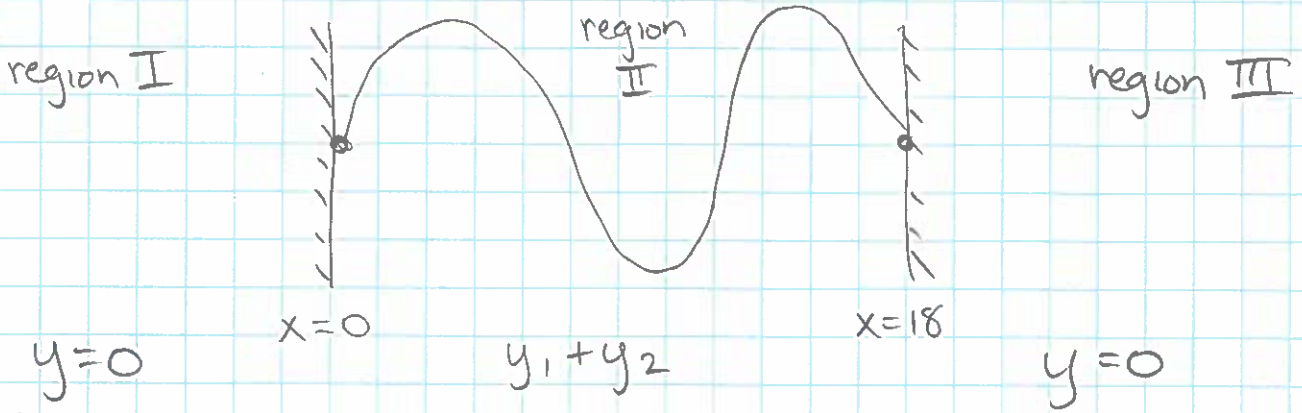
But a valid solution must also satisfy the boundary condition that the velocity of waves in one region matches velocity of waves in other region — where they meet.



The velocity of solution in region I is $V_y = 0$

The velocity of solution in region III is $V_y = 0$.

What is velocity of solution in region II?

In region II, velocity is

$$V_y = \frac{\partial y_1}{\partial t} + \frac{\partial y_2}{\partial t}$$

$$= \frac{\partial}{\partial t}\left[2\sin\left(\frac{n\pi x}{L}\right)\cos(\omega_3 t)\right]$$

$$= -\omega_3 \cdot 2\sin\left(\frac{n\pi x}{L}\right)\sin(\omega_3 t)$$

At $x=0$, $V_y = -\omega_3 \cdot 2\sin(0)\sin(\omega_3 t) = 0$

At $x=18$, $V_y = -\omega_3 \cdot 2\sin\left(\frac{3\pi}{18}18\right)\sin(\omega_3 t)$

$$= -\omega_3 \cdot 2\sin(3\pi)\sin(\omega_3 t) = 0$$

So this does match at each boundary.