$$\text{matrix } \mathbf{A} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad = \quad \begin{pmatrix} 2+4 \\ 5-2 \end{pmatrix} \quad = \quad \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad = \quad \begin{pmatrix} 2+2 \\ 5-1 \end{pmatrix} \quad = \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad = \quad 4\mathbf{x}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad = \quad \begin{pmatrix} 4+4 \\ 10-2 \end{pmatrix} \quad = \quad \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \lambda\,\mathbf{x}$$

$$\begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad = \quad \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{aligned} 2x \ + \ 2y \quad &= \quad \lambda x \\ 5x \ - \ 1y \quad &= \quad \lambda y \end{aligned}$$

$$2y \quad = \quad (\lambda - 2)x \qquad \Rightarrow \qquad y \quad = \quad \frac{1}{2}(\lambda - 2)x$$

$$5x \ - \ \frac{1}{2}\lambda x \ + \ x \quad = \quad \frac{1}{2}\lambda^2 x \ - \ \lambda x$$

$$\frac{1}{2}\lambda^2 - \frac{1}{2}\lambda - 6 = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$\lambda = 4 \quad , \quad \lambda = -3$$

$$\begin{aligned} 2x + 2y &= 4x \\ 5x - 1y &= 4y \end{aligned}$$

$$\begin{aligned} 2x &= 2y \qquad \Rightarrow \quad x = y \\ 5x &= 5y \qquad \Rightarrow \quad x = y \end{aligned}$$

eigenvector corresponding to $\lambda = 4 \quad \rightarrow \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} a \\ a \end{pmatrix}$

eigenvector corresponding to $\lambda = -3 \quad \rightarrow \quad \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ or $\begin{pmatrix} 4 \\ -10 \end{pmatrix}$ or $\begin{pmatrix} -2a \\ 5a \end{pmatrix}$

$$\frac{d^2x_1}{dt^2} + 2\frac{k}{m}x_1 - \frac{k}{m}x_2 = 0$$

$$\frac{d^2x_2}{dt^2} - \frac{k}{m}x_1 + 2\frac{k}{m}x_2 = 0$$

$$F_1 = -2kx_1 + kx_2$$

$$F_2 = kx_1 - 2kx_2 + kx_3$$

$$F_3 = kx_2 - 2kx_3$$

$$\left(\frac{k}{m}\right) \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} ax_1 \\ bx_2 \\ cx_3 \end{pmatrix} = -\omega^2 \begin{pmatrix} ax_1 \\ bx_2 \\ cx_3 \end{pmatrix}$$

$$\frac{d^2x_2}{dt^2} - \frac{k}{m}x_1 + 2\frac{k}{m}x_2 = 0$$

$$-2\frac{k}{m}x_1 + \frac{k}{m}x_2 = \frac{d^2x_1}{dt^2}$$

$$\frac{k}{m}x_1 \;-\; 2\frac{k}{m}x_2 \quad = \quad \frac{d^2x_2}{dt^2}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad = \quad \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

$$s \quad = \quad \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix}$$

$$\frac{d^2s}{dt^2} \quad = \quad -\omega^2 s$$

$$\frac{d^2x_1}{dt^2} \quad = \quad -\omega^2 x_1$$

$$\frac{d^2x_2}{dt^2} \quad = \quad -\omega^2 x_2$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix} \quad = \quad -\omega^2 \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix}$$

$$\mathbf{As} \quad = \quad \lambda\,\mathbf{s}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad -\omega^2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad \omega^2 \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \;+\; 1(\frac{k}{m})b \;+\; \omega^2 a \quad = \quad 0$$

$$1(\frac{k}{m})a \;-\; 2(\frac{k}{m})b \;+\; \omega^2 b \quad = \quad 0$$

$$b \quad = \quad \left(\frac{\omega^2 - 2\frac{k}{m}}{-\frac{k}{m}}\right) a$$

$$1(\frac{k}{m})a \;+\; \left(-2\frac{k}{m} + \omega^2\right) b \quad = \quad 0$$

$$1(\frac{k}{m})a \;+\; \left(-2\frac{k}{m} + \omega^2\right) \left(\frac{\omega^2 - 2\frac{k}{m}}{-\frac{k}{m}}\right) a \quad = \quad 0$$

$$-1\left(\frac{k}{m}\right)^2 \;+\; 4\left(\frac{k}{m}\right)^2 \;-\; 4\left(\frac{k}{m}\right)\omega^2 \;+\; \omega^4 \quad = \quad 0$$

$$\lambda \quad = \quad \omega^2$$

$$3\left(\frac{k}{m}\right)^2 \;-\; 4\left(\frac{k}{m}\right)\lambda \;+\; \lambda^2 \quad = \quad 0$$

$$\lambda \quad = \quad \omega^2 \quad = \quad 1\frac{k}{m} \quad \text{or} \quad 3\frac{k}{m}$$

$$\omega_1 \quad = \quad \sqrt{\frac{k}{m}}$$

$$\omega_2 \quad = \quad \sqrt{\frac{3k}{m}}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad 1(\frac{k}{m}) \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \;+\; 1(\frac{k}{m})b \;+\; 1(\frac{k}{m})a \quad = \quad 0$$

$$1(\frac{k}{m})a \;-\; 2(\frac{k}{m})b \;+\; 1(\frac{k}{m})b \quad = \quad 0$$

$$a \quad = \quad b$$

$$\text{eigenvalue } \omega_1 \;=\; \sqrt{\frac{k}{m}} \quad \text{corresponds to eigenvector} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_1 \quad = \quad \begin{pmatrix} 1x_1 \\ 1x_2 \end{pmatrix} \qquad \text{or} \qquad s_1 \quad = \quad x_1 \;+\; x_2$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad 3(\frac{k}{m}) \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \;+\; 1(\frac{k}{m})b \;+\; 3(\frac{k}{m})a \quad = \quad 0$$

$$1(\frac{k}{m})a \;-\; 2(\frac{k}{m})b \;+\; 3(\frac{k}{m})b \quad = \quad 0$$

$$a \quad = \quad -b$$

$$\text{eigenvalue } \omega_2 \;=\; \sqrt{\frac{3k}{m}} \quad \text{corresponds to eigenvector} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$s_2 \;=\; \begin{pmatrix} 1x_1 \\ -1x_2 \end{pmatrix} \quad \text{or} \quad s_2 \;=\; x_1 \;-\; x_2 \quad \text{or} \quad s_2 \;=\; x_2 \;-\; x_1$$

$$-\omega_1^2 \;=\; (\sqrt{2}-2)\frac{k}{m} \quad \Rightarrow \quad \omega_1 \;=\; \sqrt{(2-\sqrt{2})\frac{k}{m}} \;=\; 0.765\sqrt{\frac{k}{m}}$$

$$-\omega_2^2 \;=\; -2\frac{k}{m} \quad \Rightarrow \quad \omega_2 \;=\; \sqrt{2\frac{k}{m}} \;=\; 1.414\sqrt{\frac{k}{m}}$$

$$-\omega_3^2 \;=\; (-\sqrt{2}-2)\frac{k}{m} \quad \Rightarrow \quad \omega_3 \;=\; \sqrt{(2+\sqrt{2})\frac{k}{m}} \;=\; 1.848\sqrt{\frac{k}{m}}$$

$$s_1 \;=\; \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \;=\; x_1 \;+\; \sqrt{2}x_2 \;+\; x_3$$

$$s_2 \;=\; \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \;=\; -x_1 \;+\; 0x_2 \;+\; x_3$$

$$s_3 \;=\; \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} \;=\; x_1 \;-\; \sqrt{2}x_2 \;+\; x_3$$

$$s_1(t) \;=\; A_1\cos(\omega_1 t + \phi_1)$$

$$s_2(t) \;=\; A_2\cos(\omega_2 t + \phi_2)$$

$$s_3(t) \;=\; A_3\cos(\omega_3 t + \phi_3)$$

$$x_1 \;=\; \frac{1}{4}s_1 \;-\; \frac{1}{2}s_2 \;+\; \frac{1}{4}s_3$$

$$s_1 \;=\; \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_2 \;=\; \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\left(\frac{k}{m}\right) \begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} ax_1 \\ bx_2 \\ cx_3 \\ dx_4 \end{pmatrix} = -\omega^2 \begin{pmatrix} ax_1 \\ bx_2 \\ cx_3 \\ dx_4 \end{pmatrix}$$

$$\omega_1^2 = 0.38197\frac{k}{m} \quad \Rightarrow \quad \omega_1 = 0.618\sqrt{\frac{k}{m}}$$

$$\omega_2^2 = 3.61803\frac{k}{m} \quad \Rightarrow \quad \omega_2 = 1.902\sqrt{\frac{k}{m}}$$

$$\omega_3^2 = 2.61803\frac{k}{m} \quad \Rightarrow \quad \omega_3 = 1.618\sqrt{\frac{k}{m}}$$

$$\omega_4^2 = 1.38197\frac{k}{m} \quad \Rightarrow \quad \omega_4 = 1.176\sqrt{\frac{k}{m}}$$

$$s_1 = \begin{pmatrix} 1 \\ 1.61803 \\ 1.61803 \\ 1 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} -1 \\ -0.61803 \\ 0.61803 \\ 1 \end{pmatrix}$$

$$s_3 = \begin{pmatrix} 1 \\ -0.61803 \\ -0.61803 \\ 1 \end{pmatrix}$$

$$s_4 = \begin{pmatrix} -1 \\ 1.61803 \\ -1.61803 \\ 1 \end{pmatrix}$$

$$\omega_n = 2\left(\sqrt{\frac{k}{m}}\right) \sin\left[\frac{n\pi}{2(N+1)}\right] = 2\,\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right]$$

$$n \quad << \quad N$$

$$\left[\frac{n\pi}{2(N+1)}\right] \quad << \quad 1$$

$$\omega_n \quad \simeq \quad 2\left(\sqrt{\frac{k}{m}}\right)\left[\frac{n\pi}{2(N+1)}\right] \quad \simeq \quad 2\,\omega_0\left[\frac{n\pi}{2(N+1)}\right]$$

$$\omega_n \quad \simeq \quad \left(2\,\omega_0\left[\frac{\pi}{2(N+1)}\right]\right)\ n$$

$$\omega_1 \quad \simeq \quad 2\,\omega_0\left[\frac{\pi}{2(N+1)}\right]$$

$$\omega_n \quad \simeq \quad n\,\omega_1$$

$$\frac{F}{\Delta L} \quad = \quad \frac{A}{L}\,Y$$

$$\frac{F}{\Delta L} \quad = \quad \frac{A}{L}\,Y \qquad\qquad \frac{F}{\Delta x} \quad = \quad k$$

$$\omega_0 \quad = \quad \sqrt{\frac{k}{m}}$$

$$\omega_0 \quad = \quad \sqrt{\frac{AY/L}{\rho AL}}$$

$$f_0 \quad = \quad \frac{\omega_0}{2\pi} \quad = \quad 2.4\times 10^{12}\mathrm{Hz}$$

$$\lambda_0 \quad = \quad \frac{c}{f_0} \quad = \quad 122\ \mu\mathrm{m}$$

$$\omega_N \quad = \quad 2\omega_0 \qquad \Rightarrow \qquad \lambda_N \quad = \quad \frac{1}{2}\ \lambda_0$$