$$n \quad << \quad N$$

$$\left[\frac{n\pi}{2(N+1)}\right] \quad << \quad 1$$

$$\omega_n \quad \simeq \quad 2\left(\sqrt{\frac{k}{m}}\right)\left[\frac{n\pi}{2(N+1)}\right] \quad \simeq \quad 2\,\omega_0\left[\frac{n\pi}{2(N+1)}\right]$$

$$\omega_n \quad \simeq \quad n\,\omega_1$$

$$\frac{F}{\Delta L} \quad = \quad \frac{A}{L}\,Y$$

$$\frac{F}{\Delta L} \quad = \quad \frac{A}{L}\,Y \qquad\qquad \frac{F}{\Delta x} \quad = \quad k$$

$$\omega_0 \quad = \quad \sqrt{\frac{k}{m}}$$

$$\omega_0 \quad = \quad \sqrt{\frac{AY/L}{\rho AL}}$$

$$f_0 \quad = \quad \frac{\omega_0}{2\pi} \quad = \quad 2.4\times 10^{12}\mathrm{Hz}$$

$$\lambda_0 \quad = \quad \frac{c}{f_0} \quad = \quad 122\ \mu\mathrm{m}$$

$$\omega_N \quad = \quad 2\omega_0 \qquad \Rightarrow \qquad \lambda_N \quad = \quad \frac{1}{2}\,\lambda_0$$

$$\omega_0 \quad = \quad \frac{1}{L}\sqrt{\frac{Y}{\rho}}$$

$$\omega_N \quad \simeq \quad 3\times 10^{13}\ \mathrm{rad/s}$$

$$\omega_N \quad \sim \quad 2\sqrt{\frac{k}{m_{\mathrm{avg}}}}$$

$$F_1 \quad = \quad k(x_2 - x_1) \quad = \quad -kx_1 \ + \ kx_2$$

$$F_2 \quad = \quad -k(x_2 - x_1) \quad = \quad kx_1 \ - \ kx_2$$

$$F_1 = m_C \frac{d^2 x_1}{dt^2}$$

$$F_2 = m_O \frac{d^2 x_2}{dt^2}$$

$$F_1 = \left(\frac{3}{4} m_O\right) \frac{d^2 x_1}{dt^2}$$

$$F_2 = m_O \frac{d^2 x_2}{dt^2}$$

$$-kx_1 + kx_2 = \left(\frac{3}{4} m_O\right) \frac{d^2 x_1}{dt^2}$$

$$kx_1 - kx_2 = m_O \frac{d^2 x_2}{dt^2}$$

$$\left(\frac{k}{m_O}\right) \begin{pmatrix} -\frac{4}{3} & \frac{4}{3} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$-\frac{4}{3}\frac{k}{m_o} a + \frac{4}{3}\frac{k}{m_o} b = -\omega^2 a$$

$$b = \frac{\left(\frac{4}{3}\frac{k}{m_o} - \omega^2\right) a}{\frac{4}{3}\frac{k}{m_o}}$$

$$\frac{4}{3}\left(\frac{k}{m_o}\right)^2 - \frac{4}{3}\left(\frac{k}{m_o}\right)^2 + \frac{k}{m_o}\omega^2 + \frac{4}{3}\frac{k}{m_o}\omega^2 - \omega^4 = 0$$

$$\frac{7}{3}\frac{k}{m_o}\omega^2 - \omega^4 = 0$$

$$\omega^2 = \frac{7}{3}\frac{k}{m}$$

$$k = \frac{3}{7} m_o \omega^2$$

$$\begin{aligned} F_1 &= k(x_2 - x_1) &&= -kx_1 + kx_2 \\ F_2 &= -k(x_2 - x_1) + k(x_3 - x_2) &&= kx_1 - 2kx_2 + kx_3 \\ F_3 &= -k(x_3 - x_2) &&= kx_2 - kx_3 \end{aligned}$$

$$m_o \frac{d^2 x_1}{dt^2} = -kx_1 + kx_2 + 0x_3$$

$$m_c \frac{d^2 x_2}{dt^2} = kx_1 - 2kx_2 + kx_3$$

$$m_o \frac{d^2 x_3}{dt^2} = 0x_1 + kx_2 - kx_3$$

$$\begin{pmatrix} -\frac{k}{m_o} & \frac{k}{m_o} & 0 \\ \frac{k}{m_c} & -2\frac{k}{m_c} & \frac{k}{m_c} \\ 0 & \frac{k}{m_o} & -\frac{k}{m_o} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{d^2 x_1}{dt^2} \\ \frac{d^2 x_2}{dt^2} \\ \frac{d^2 x_3}{dt^2} \end{pmatrix}$$

$$m_c = \frac{3}{4}\, m_o$$

$$\begin{pmatrix} -\frac{k}{m_o} & \frac{k}{m_o} & 0 \\ \frac{4}{3}\frac{k}{m_o} & -\frac{8}{3}\frac{k}{m_o} & \frac{4}{3}\frac{k}{m_o} \\ 0 & \frac{k}{m_o} & -\frac{k}{m_o} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{d^2 x_1}{dt^2} \\ \frac{d^2 x_2}{dt^2} \\ \frac{d^2 x_3}{dt^2} \end{pmatrix}$$

$$\left(\frac{k}{m_o}\right) \begin{pmatrix} -1 & 1 & 0 \\ \frac{4}{3} & -\frac{8}{3} & \frac{4}{3} \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{d^2 x_1}{dt^2} \\ \frac{d^2 x_2}{dt^2} \\ \frac{d^2 x_3}{dt^2} \end{pmatrix}$$

$$\left(\frac{k}{m_o}\right) \begin{pmatrix} -1 & 1 & 0 \\ \frac{4}{3} & -\frac{8}{3} & \frac{4}{3} \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\omega^2 \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\omega_1^2 = 0\, \frac{k}{m_o} = 0$$

$$\omega_2^2 = \frac{k}{m_o}$$

$$\omega_3^2 = \left(\frac{11}{3}\right) \frac{k}{m_o}$$

$$
\begin{aligned}
\omega_2 &= \sqrt{\frac{k}{m_o}} & \Rightarrow \qquad s_2 &= \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \\
\omega_3 &= \sqrt{\frac{11k}{3m_o}} & \Rightarrow \qquad s_3 &= \begin{pmatrix} 1 \\ -8/3 \\ 1 \end{pmatrix}
\end{aligned}
$$