$$\text{impedence Z} \quad = \quad \frac{\text{force on string in y direction}}{\text{amplitude of velocity in y direction}}$$

$$Z \quad = \quad \frac{F_y}{v_y}$$

$$F_y \quad = \quad T_y \quad = \quad T\,\sin(\theta) \quad = \quad T\left(\frac{T_y}{\sqrt{T_x^2+T_y^2}}\right)$$

$$\sqrt{T_x^2+T_y^2} \quad \simeq \quad T_x$$

$$F_y \quad \simeq \quad T\left(\frac{T_y}{T_x}\right) \quad = \quad T\,(\text{slope at P}) \quad = \quad T\,\frac{\partial y}{\partial x}$$

$$F_y \quad = \quad -T\,\frac{\partial y}{\partial x}$$

$$y(x,t) \quad = \quad A_i\,\sin(k_I x - \omega t)$$

$$v_y \quad = \quad \frac{\partial y}{\partial t} \quad = \quad -\omega A_i\,\cos(k_I x - \omega t)$$

$$F_y \quad = \quad -T\Big[k_I A_i \cos\left(k_I x - \omega t\right)\Big]$$

$$Z_I \quad = \quad \frac{F_y}{v_y} \quad = \quad \frac{-T k_I A_i\,\cos(k_I x - \omega t)}{-\omega A_i\,\cos(k_I x - \omega t)}$$

$$Z_I \quad = \quad \frac{T k_I}{\omega}$$

$$Z_I \quad = \quad \frac{T}{v_I}$$

$$v_I \quad = \quad \sqrt{\frac{T}{\mu_I}}$$

$$Z_I \quad = \quad \sqrt{T\mu_I}$$

$$A_r \quad = \quad \left(\frac{Z_{II}-Z_I}{Z_I+Z_{II}}\right)A_i$$

$$A_t \quad = \quad \left(\frac{2\,Z_I}{Z_I+Z_{II}}\right)A_i$$

$$Z_{II} = Z_I$$

$$Z_I = \sqrt{T_I \mu_I} = \sqrt{T_{II} \mu_{II}} = Z_{II}$$

$$F = -bv \qquad \Rightarrow \qquad b = \frac{F}{v}$$

$$b = Z_I$$

$$Z = \frac{V}{I}$$