$$\omega_0 \quad = \quad \sqrt{\frac{k}{m}}$$

$$\vec{F}(\text{air}) \quad = \quad -b\vec{v}$$

$$F(t) \quad = \quad F_{\max}\, e^{i\omega t}$$

$$m\frac{d^2x}{dt^2} \; + \; b\frac{dx}{dt} \; + \; kx \quad = \quad F_{\max}\, e^{i\omega t}$$

$$\frac{d^2x}{dt^2} \; + \; \frac{b}{m}\frac{dx}{dt} \; + \; \frac{k}{m}x \quad = \quad 0$$

$$x(t) \quad = \quad Ae^{i(\omega t+\phi)}$$

$$\frac{dx}{dt} \quad = \quad i\omega Ae^{i(\omega t+\phi)} \quad = \quad i\omega\, x(t)$$

$$\frac{d^2x}{dt^2} \quad = \quad -\omega^2 Ae^{i(\omega t+\phi)} \quad = \quad -\omega^2\, x(t)$$

$$-m\omega^2 Ae^{i(\omega t+\phi)} \; + \; ib\omega Ae^{i(\omega t+\phi)} \; + \; kAe^{i(\omega t+\phi)} \quad = \quad F_{\max}e^{i\omega t}$$

$$-m\omega^2 Ae^{i\phi} \; + \; ib\omega Ae^{i\phi} \; + \; kAe^{i\phi} \quad = \quad F_{\max}$$

$$-\omega^2 Ae^{i\phi} \; + \; i\frac{b\omega}{m}Ae^{i\phi} \; + \; \frac{k}{m}Ae^{i\phi} \quad = \quad \frac{F_{\max}}{m}$$

$$-\omega^2 Ae^{i\phi} \; + \; i\frac{b\omega}{m}Ae^{i\phi} \; + \; {\omega_0}^2 Ae^{i\phi} \quad = \quad \frac{F_{\max}}{m}$$

$$-\omega^2 A \; + \; i\frac{b\omega}{m}A \; + \; {\omega_0}^2 A \quad = \quad \frac{F_{\max}}{m}e^{-i\phi}$$

$$-\omega^2 A \; + \; i\frac{b\omega}{m}A \; + \; {\omega_0}^2 A \quad = \quad \frac{F_{\max}}{m}\; [\cos(\phi) - i\sin(\phi)]$$

$$-\omega^2 A \; + \; {\omega_0}^2 A \quad = \quad ({\omega_0}^2 - \omega^2)A \quad = \quad \frac{F_{\max}}{m}\; \cos(\phi)$$

$$\frac{b\omega}{m}A \quad = \quad -\frac{F_{\max}}{m}\; \sin(\phi)$$

$$\cos(\phi) \quad = \quad \frac{Am(\omega_0^2 - \omega^2)}{F_{\max}}$$

$$\sin(\phi) \quad = \quad -\frac{Ab\,\omega}{F_{\max}}$$

$$F_{\max} \quad = \quad \sqrt{A^2 m^2 (\omega_0^2 - \omega^2)^2 \; + \; A^2 b^2 \omega^2}$$

$$F_{\max} \quad = \quad A\sqrt{m^2 (\omega_0^2 - \omega^2)^2 \; + \; b^2 \omega^2}$$

$$A \quad = \quad \frac{F_{\max}}{\sqrt{m^2 (\omega_0^2 - \omega^2)^2 \; + \; b^2 \omega^2}}$$

$$A \quad = \quad \frac{F_{\max}/m}{\sqrt{(\omega_0^2 - \omega^2)^2 \; + \; \frac{b^2}{m^2}\omega^2}}$$

$$\phi \quad = \quad \tan^{-1}\left(\frac{(\frac{b}{m})\,\omega}{\omega_0^2 - \omega^2}\right)$$

$$\frac{d^2x}{dt^2} \; + \; \frac{b}{m}\frac{dx}{dt} \; + \; \frac{k}{m}x \quad = \quad 0$$

$$x(t) \quad = \quad Ae^{i(\omega t + \phi) - (t/\tau)}$$

$$\tau \quad = \quad \frac{2m}{b}$$

$$\mathrm{SPE}(x) \quad = \quad \frac{1}{2}k\,[x(t)]^2 \quad = \quad \frac{1}{2}k\,A^2 e^{2i[\omega t + \phi] - (2t/\tau)}$$

$$\text{amplitude time constant} \quad \tau \quad = \quad \frac{2m}{b}$$

$$\text{energy time constant} \quad \frac{\tau}{2} \quad = \quad \frac{m}{b}$$

$$\text{time to move one cycle} \quad P \quad = \quad \frac{2\pi}{\omega_0}$$

$$\text{time to move one radian} \quad \frac{P}{2\pi} \quad = \quad \frac{1}{\omega_0}$$

$$Q \quad \equiv \quad \frac{\tau/2}{1/\omega_0} \quad = \quad \frac{\omega_0 m}{b}$$

$$Q \quad = \quad \frac{\tau}{2\omega_0}$$

$$\text{amplitude} \quad \propto \quad e^{-\frac{t}{\tau}}$$

$$\text{decay in amplitude in one cycle} \quad = \quad e^{-(\pi/Q)} \quad = \quad \left(\frac{1}{e}\right)^{(\pi/Q)}$$

$$\text{decay in energy in one cycle} \quad = \quad e^{-(2\pi/Q)} \quad = \quad \left(\frac{1}{e}\right)^{(2\pi/Q)}$$

$$\Delta x_{\text{static}} \quad = \quad \frac{F_{\max}}{k}$$

$$A \quad = \quad \frac{F_{\max}/m}{\left(\frac{b}{m}\right)\omega_0}$$

$$Q \quad = \quad \frac{\omega_0\, m}{b}$$

$$A \quad = \quad \frac{F_{\max}}{kQ} \quad = \quad Q\,\left(\frac{F_{\max}}{k}\right)$$

$$A \quad = \quad \frac{F_{\max}/m}{\sqrt{\left(\omega_0^2-\omega^2\right)^2 \;+\; \frac{\omega_0^2}{Q^2}\,\omega^2}}$$

$$\text{Power dissipated} \quad \propto \quad A^2 \quad \propto \quad \frac{F_{\max}^2/m^2}{\left(\omega_0^2-\omega^2\right)^2 \;+\; \frac{\omega_0^2}{Q^2}\,\omega^2}$$

$$\text{Peak power dissipated} \quad \propto \quad \frac{F_{\max}^2/m^2}{\frac{\omega_0^2}{Q^2}\,\omega^2}$$

$$\left(\omega_0^2-\omega^2\right)^2 \;+\; \frac{\omega_0^2}{Q^2}\,\omega^2 \quad = \quad 2\;\frac{\omega_0^2}{Q^2}\,\omega^2$$

$$\left(\omega_0^2-\omega^2\right) \quad = \quad \frac{\omega_0}{Q}\,\omega$$

$$(\omega_0+\omega)(\omega_0-\omega) \quad = \quad \frac{\omega_0}{Q}\,\omega$$

$$2\omega_0(\omega_0-\omega) \quad \simeq \quad \frac{\omega_0}{Q}\,\omega$$

$$(\omega_0-\omega) \quad \simeq \quad \frac{\omega_0}{2Q}$$

$$\frac{\Delta\omega}{\omega_0} \simeq \frac{1}{2Q}$$

$$x(t) = Ae^{-t/\tau}$$

$$x(0) = Ae^0 = A$$

$$v(t) = -\frac{1}{\tau}Ae^{-t/\tau}$$

$$v(0) = -\frac{1}{\tau}Ae^0 = -\frac{1}{\tau}A$$

$$x(t) = Ae^{-t/\tau} + Bte^{-t/\tau}$$

$$v(t) = -\frac{1}{\tau}Ae^{-t/\tau} + Be^{-t/\tau} - \frac{1}{\tau}Bte^{-t/\tau}$$

$$b = 2m\omega_0 = 2\sqrt{km}$$

$$f(t) = f(t+P)$$

$$A_n \cos(\omega_n t + \phi_n)$$

$$A_n \cos(\omega_n t) + B_n \sin(\omega_n t)$$

$$\omega_1 = \frac{2\pi}{P}$$

$$\omega_2 = 2\omega_1 = \frac{2\pi(2)}{P}$$

$$\omega_3 = 3\omega_1 = \frac{2\pi(3)}{P}$$

$$\omega_n = n\omega_1 = \frac{2\pi n}{P}$$

$$\begin{aligned} B_1^2 \int_0^P \sin^2(\omega_1 t)dt &= \frac{1}{2}B_1^2 \int_0^P 1 - \cos(2\omega_1 t)dt \\ &= \frac{1}{2}B_1^2 \int_0^P dt - \frac{1}{2}B_1^2 \int_0^P \cos(2\omega_1 t)dt \end{aligned}$$

$$\begin{aligned}
\text{if } n > 0, \quad A_n &= \frac{2}{P}\int_0^P f(t)\cos(\omega_n t)dt \\
B_n &= \frac{2}{P}\int_0^P f(t)\sin(\omega_n t)dt
\end{aligned}$$

$$\begin{aligned}
\text{if } n = 0, \quad A_0 &= \frac{1}{P}\int_0^P f(t)dt \\
B_0 &= 0
\end{aligned}$$

$$\begin{aligned}
\text{if } n > 0, \quad A_n &= \frac{2}{P}\int_0^P f(t)\cos\left(\frac{2\pi n}{P}t\right)dt \\
B_n &= \frac{2}{P}\int_0^P f(t)\sin\left(\frac{2\pi n}{P}t\right)dt
\end{aligned}$$

$$\begin{aligned}
f(t) \simeq\ & -0.636\ \sin\left(\frac{2\pi t}{10\text{ s}}\right) - 0.318\ \sin\left(\frac{4\pi t}{10\text{ s}}\right) - 0.212\ \sin\left(\frac{6\pi t}{10\text{ s}}\right) \\
& - 0.158\ \sin\left(\frac{8\pi t}{10\text{ s}}\right) - 0.126\ \sin\left(\frac{10\pi t}{10\text{ s}}\right)
\end{aligned}$$