$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\vec{F}(\text{air}) = -b\vec{v}$$

$$F(t) = F_{\text{max}}\, e^{i\omega t}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{\text{max}}\, e^{i\omega t}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$x(t) = Ae^{i(\omega t+\phi)}$$

$$\frac{dx}{dt} = i\omega Ae^{i(\omega t+\phi)} = i\omega\, x(t)$$

$$\frac{d^2x}{dt^2} = -\omega^2 Ae^{i(\omega t+\phi)} = -\omega^2\, x(t)$$

$$-m\omega^2 Ae^{i(\omega t+\phi)} + ib\omega Ae^{i(\omega t+\phi)} + kAe^{i(\omega t+\phi)} = F_{\text{max}}e^{i\omega t}$$

$$-m\omega^2 Ae^{i\phi} + ib\omega Ae^{i\phi} + kAe^{i\phi} = F_{\text{max}}$$

$$-\omega^2 Ae^{i\phi} + i\frac{b\omega}{m}Ae^{i\phi} + \frac{k}{m}Ae^{i\phi} = \frac{F_{\text{max}}}{m}$$

$$-\omega^2 Ae^{i\phi} + i\frac{b\omega}{m}Ae^{i\phi} + {\omega_0}^2 Ae^{i\phi} = \frac{F_{\text{max}}}{m}$$

$$-\omega^2 A + i\frac{b\omega}{m}A + {\omega_0}^2 A = \frac{F_{\text{max}}}{m}e^{-i\phi}$$

$$-\omega^2 A + i\frac{b\omega}{m}A + {\omega_0}^2 A = \frac{F_{\text{max}}}{m}\,[\cos(\phi) - i\sin(\phi)]$$

$$-\omega^2 A + {\omega_0}^2 A = ({\omega_0}^2 - \omega^2)A = \frac{F_{\text{max}}}{m}\,\cos(\phi)$$

$$\frac{b\omega}{m}A = -\frac{F_{\text{max}}}{m}\,\sin(\phi)$$

$$\cos(\phi) = \frac{Am(\omega_0^2 - \omega^2)}{F_{\max}}$$

$$\sin(\phi) = -\frac{Ab\,\omega}{F_{\max}}$$

$$F_{\max} = \sqrt{A^2 m^2 \left(\omega_0^2 - \omega^2\right)^2 \ + \ A^2 b^2 \omega^2}$$

$$F_{\max} = A\sqrt{m^2 \left(\omega_0^2 - \omega^2\right)^2 \ + \ b^2 \omega^2}$$

$$A = \frac{F_{\max}}{\sqrt{m^2 \left(\omega_0^2 - \omega^2\right)^2 \ + \ b^2 \omega^2}}$$

$$A = \frac{F_{\max}/m}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 \ + \ \frac{b^2}{m^2}\omega^2}}$$

$$\phi = \tan^{-1}\left(\frac{(\frac{b}{m})\,\omega}{\omega_0^2 - \omega^2}\right)$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$x(t) = Ae^{i(\omega t + \phi) - (t/\tau)}$$

$$\tau = \frac{2m}{b}$$

$$\mathrm{SPE}(x) = \frac{1}{2}k\,[x(t)]^2 = \frac{1}{2}k\,A^2 e^{2i[\omega t + \phi] - (2t/\tau)}$$

$$\text{amplitude time constant} \quad \tau = \frac{2m}{b}$$

$$\text{energy time constant} \quad \frac{\tau}{2} = \frac{m}{b}$$

$$\text{time to move one cycle} \quad P = \frac{2\pi}{\omega_0}$$

$$\text{time to move one radian} \quad \frac{P}{2\pi} = \frac{1}{\omega_0}$$

$$Q \equiv \frac{\tau/2}{1/\omega_0} = \frac{\omega_0 m}{b}$$

$$Q = \frac{\tau}{2\omega_0}$$

$$\text{amplitude} \propto e^{-\frac{t}{\tau}}$$

$$\text{decay in amplitude in one cycle} = e^{-(\pi/Q)} = \left(\frac{1}{e}\right)^{(\pi/Q)}$$

$$\text{decay in energy in one cycle} = e^{-(2\pi/Q)} = \left(\frac{1}{e}\right)^{(2\pi/Q)}$$

$$\Delta x_{\text{static}} = \frac{F_{\text{max}}}{k}$$

$$A = \frac{F_{\text{max}}/m}{\left(\frac{b}{m}\right)\omega_0}$$

$$Q = \frac{\omega_0\, m}{b}$$

$$A = \frac{F_{\text{max}}}{kQ} = Q\left(\frac{F_{\text{max}}}{k}\right)$$

$$A = \frac{F_{\text{max}}/m}{\sqrt{\left(\omega_0^2-\omega^2\right)^2 + \frac{\omega_0^2}{Q^2}\,\omega^2}}$$

$$\text{Power dissipated} \propto A^2 \propto \frac{F_{\text{max}}^2/m^2}{\left(\omega_0^2-\omega^2\right)^2 + \frac{\omega_0^2}{Q^2}\,\omega^2}$$

$$\text{Peak power dissipated} \propto \frac{F_{\text{max}}^2/m^2}{\frac{\omega_0^2}{Q^2}\,\omega^2}$$

$$\left(\omega_0^2-\omega^2\right)^2 + \frac{\omega_0^2}{Q^2}\,\omega^2 = 2\,\frac{\omega_0^2}{Q^2}\,\omega^2$$

$$\left(\omega_0^2-\omega^2\right) = \frac{\omega_0}{Q}\,\omega$$

$$(\omega_0+\omega)(\omega_0-\omega) = \frac{\omega_0}{Q}\,\omega$$

$$2\omega_0(\omega_0-\omega) \simeq \frac{\omega_0}{Q}\,\omega$$

$$(\omega_0-\omega) \simeq \frac{\omega_0}{2Q}$$

$$\frac{\Delta\omega}{\omega_0} \simeq \frac{1}{2Q}$$

$$x(t) = Ae^{-t/\tau}$$

$$x(0) = Ae^{0} = A$$

$$v(t) = -\frac{1}{\tau}Ae^{-t/\tau}$$

$$v(0) = -\frac{1}{\tau}Ae^{0} = -\frac{1}{\tau}A$$

$$x(t) = Ae^{-t/\tau} + Bte^{-t/\tau}$$

$$v(t) = -\frac{1}{\tau}Ae^{-t/\tau} + Be^{-t/\tau} - \frac{1}{\tau}Bte^{-t/\tau}$$