$$\text{matrix } \mathbf{A} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad = \quad \begin{pmatrix} 2+4 \\ 5-2 \end{pmatrix} \quad = \quad \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad = \quad \begin{pmatrix} 2+2 \\ 5-1 \end{pmatrix} \quad = \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad = \quad 4\mathbf{x}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad = \quad \begin{pmatrix} 4+4 \\ 10-2 \end{pmatrix} \quad = \quad \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \lambda\,\mathbf{x}$$

$$\begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \quad = \quad \lambda\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{aligned} 2x \ + \ 2y \quad &= \quad \lambda x \\ 5x \ - \ 1y \quad &= \quad \lambda y \end{aligned}$$

$$2y \quad = \quad (\lambda-2)x \qquad \Rightarrow \qquad y \quad = \quad \frac{1}{2}(\lambda-2)x$$

$$5x \ - \ \frac{1}{2}\lambda x \ + \ x \quad = \quad \frac{1}{2}\lambda^2 x \ - \ \lambda x$$

$$\frac{1}{2}\lambda^2 - \frac{1}{2}\lambda - 6 = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$\lambda = 4 \quad , \quad \lambda = -3$$

$$\begin{aligned} 2x + 2y &= 4x \\ 5x - 1y &= 4y \end{aligned}$$

$$\begin{aligned} 2x &= 2y \qquad \Rightarrow \quad x = y \\ 5x &= 5y \qquad \Rightarrow \quad x = y \end{aligned}$$

$$\text{eigenvector corresponding to } \lambda = 4 \quad \rightarrow \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ a \end{pmatrix}$$

$$\text{eigenvector corresponding to } \lambda = -3 \quad \rightarrow \quad \begin{pmatrix} -2 \\ 5 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 4 \\ -10 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -2a \\ 5a \end{pmatrix}$$

$$\frac{d^2x_1}{dt^2} + 2\frac{k}{m}x_1 - \frac{k}{m}x_2 = 0$$

$$\frac{d^2x_2}{dt^2} - \frac{k}{m}x_1 + 2\frac{k}{m}x_2 = 0$$

$$-2\frac{k}{m}x_1 + \frac{k}{m}x_2 = \frac{d^2x_1}{dt^2}$$

$$\frac{k}{m}x_1 - 2\frac{k}{m}x_2 = \frac{d^2x_2}{dt^2}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

$$s = \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix}$$

$$\frac{d^2s}{dt^2} = -\omega^2 s$$

$$\frac{d^2x_1}{dt^2} = -\omega^2 x_1$$

$$\frac{d^2x_2}{dt^2} \quad = \quad -\omega^2 x_2$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix} \quad = \quad -\omega^2 \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix}$$

$$\mathbf{As} \quad = \quad \lambda\,\mathbf{s}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad -\omega^2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad \omega^2 \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \;+\; 1(\frac{k}{m})b \;+\; \omega^2 a \quad = \quad 0$$

$$1(\frac{k}{m})a \;-\; 2(\frac{k}{m})b \;+\; \omega^2 b \quad = \quad 0$$

$$b \quad = \quad \left(\frac{\omega^2 - 2\frac{k}{m}}{-\frac{k}{m}}\right) a$$

$$1(\frac{k}{m})a \;+\; \left(-2\frac{k}{m} + \omega^2\right) b \quad = \quad 0$$

$$1(\frac{k}{m})a \;+\; \left(-2\frac{k}{m} + \omega^2\right) \left(\frac{\omega^2 - 2\frac{k}{m}}{-\frac{k}{m}}\right) a \quad = \quad 0$$

$$-1\left(\frac{k}{m}\right)^2 \;+\; 4\left(\frac{k}{m}\right)^2 \;-\; 4\left(\frac{k}{m}\right)\omega^2 \;+\; \omega^4 \quad = \quad 0$$

$$\lambda \quad = \quad \omega^2$$

$$3\left(\frac{k}{m}\right)^2 \;-\; 4\left(\frac{k}{m}\right)\lambda \;+\; \lambda^2 \quad = \quad 0$$

$$\lambda \quad = \quad \omega^2 \quad = \quad 1\frac{k}{m} \quad \text{or} \quad 3\frac{k}{m}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad 1(\frac{k}{m}) \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \;+\; 1(\frac{k}{m})b \;+\; 1(\frac{k}{m})a \quad = \quad 0$$

$$1(\frac{k}{m})a \;-\; 2(\frac{k}{m})b \;+\; 1(\frac{k}{m})b \quad = \quad 0$$

$$a \quad = \quad b$$

$$\text{eigenvalue } \omega_1 \;=\; \sqrt{\frac{k}{m}} \quad \text{corresponds to eigenvector} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_1 \quad = \quad \begin{pmatrix} 1x \\ 1y \end{pmatrix} \qquad \text{or} \qquad s_1 \quad = \quad x_1 \;+\; x_2$$