$$y_p(t) = C \sin\left(\frac{pn\pi}{N+1}\right)\cos(\omega t)$$

$$L_{\text{tot}} = NL$$

$$n = 1 \qquad \lambda_1 = \frac{2L_{\text{tot}}}{1}$$

$$n = 2 \qquad \lambda_2 = \frac{2L_{\text{tot}}}{2}$$

$$n = 3 \qquad \lambda_3 = \frac{2L_{\text{tot}}}{3}$$

$$n = n \qquad \lambda_n = \frac{2L_{\text{tot}}}{n}$$

$$\omega_n = \sqrt{\frac{4T}{mL}}\ \sin\left(\frac{n\pi}{2(N+1)}\right)$$

$$N >> 1$$

$$n << N$$

$$\frac{n\pi}{2(N+1)} << 1$$

$$\sin\left(\frac{n\pi}{2(N+1)}\right) \simeq \frac{n\pi}{2(N+1)} = \left(\frac{\pi}{2(N+1)}\right) n$$

$$\omega_n \simeq \sqrt{\frac{4T}{mL}}\ \left(\frac{\pi}{2(N+1)}\right) n$$

$$v = \lambda f = \lambda \frac{\omega}{2\pi}$$

$$v_n = \left(\frac{2L_{\text{tot}}}{n}\right)\ \frac{1}{2\pi}\ \sqrt{\frac{4T}{mL}}\ \left(\frac{\pi}{2(N+1)}\right)\ n$$

$$n \sim \frac{1}{2}N$$

$$\sin\left(\frac{n\pi}{2(N+1)}\right) \neq \frac{n\pi}{2(N+1)}$$

$$\omega_p \simeq 56 \times 10^3 \sqrt{n_e} \quad \text{rad/s}$$

$$v_{pl} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

$$v_{pl} = c\sqrt{1 - \frac{9.4 \times 10^7}{\omega^2}}$$

$$t_0 = \frac{D}{c} = 6500 \text{ years} \simeq 2 \times 10^{11} \text{sec}$$

$$t_2 - t_1 = \Delta t = \frac{D}{c}\left(\qquad - \qquad \right)$$

$$\Delta t = \frac{D}{c}\left(\qquad \right)$$

$$D = \frac{c\Delta t}{\left(\qquad \right)}$$