$$
\begin{aligned}
\cos(\theta) \quad &= \quad \cos(0) \ + \ \frac{\theta^1}{1!}(-\sin(0)) \ + \ \frac{\theta^2}{2!}(-\cos(0)) \ + \ \frac{\theta^3}{3!}(\sin(0)) \ + \ \frac{\theta^4}{4!}(\cos(0)) \ + \ ... \\
&= \quad 1 \quad + \quad 0 \quad - \quad \frac{\theta^2}{2!} \quad - \quad 0 \quad + \quad \frac{\theta^4}{4!} \quad + \quad ...
\end{aligned}
$$

$$
\begin{aligned}
\sin(\theta) \quad &= \quad \sin(0) \ + \ \frac{\theta^1}{1!}(\cos(0)) \ + \ \frac{\theta^2}{2!}(-\sin(0)) \ + \ \frac{\theta^3}{3!}(-\cos(0)) \ + \ \frac{\theta^4}{4!}(\sin(0)) \ + ... \\
&= \quad 0 \quad + \quad \frac{\theta^1}{1!} \quad - \quad 0 \quad - \quad \frac{\theta^3}{3!} \quad + \quad 0 \quad + \quad \frac{\theta^5}{5!} \quad + \quad ...
\end{aligned}
$$

$$
\begin{aligned}
e^{i\theta} \quad &= \quad e^0 \ + \ \frac{i\theta^1}{1!}(e^0) \ + \ \frac{i^2\theta^2}{2!}(e^0) \ + \ \frac{i^3\theta^3}{3!}(e^0) \ + \ \frac{i^4\theta^4}{4!}(e^0) \ + ... \\
&= \quad 1 \quad + \quad \frac{i\theta^1}{1!} \quad - \quad \frac{\theta^2}{2!} \quad - \quad \frac{i\theta^3}{3!} \quad + \quad \frac{\theta^4}{4!} \quad + \quad \frac{i\theta^5}{5!} \quad + \quad ...
\end{aligned}
$$

$$\frac{d^2y}{dt^2} \quad = \quad -\left(\frac{k}{m}\right) y \quad = \quad -\,\omega^2 y$$

$$\omega \quad \equiv \quad \sqrt{\frac{k}{m}}$$

$$y(t) \quad = \quad A\cos(\omega t + \phi)$$

$$y(t) \quad = \quad B\cos(\omega t) \ + \ C\sin(\omega t)$$

$$y(0) \quad = \quad 0 \text{ m}$$

$$\frac{dy}{dt}|_{t=0} \ = \ v_y(0) \quad = \quad 0.12 \text{ m/s}$$

$$y(t) \quad = \quad (-0.04 \text{ m})\cos(\omega t + \frac{\pi}{2})$$

$$y(t) \quad = \quad (0) \ \cos(\omega t) \ + \ (0.04 \text{ m}) \ \sin(\omega t)$$

$$y(t) \quad = \quad Ae^{i(\omega t+\phi)}$$

$$e^{i\theta} \quad = \quad \cos(\theta) \ + \ i\,\sin(\theta)$$

$$y(t) \quad = \quad Ae^{i(\omega t + \phi)} \quad = \quad A\cos(\omega t + \phi) \; + \; i\,A\sin(\omega t + \phi)$$

$$\frac{dy}{dt} \quad = \quad v_y(t) \quad = \quad i\omega A e^{i(\omega t + \phi)} \quad = \quad i\omega y(t)$$

$$\frac{d^2y}{dt^2} \quad = \quad a_y(t) \quad = \quad -\omega^2 A e^{i(\omega t + \phi)} \quad = \quad -\omega^2 y(t)$$

$$y(t) \quad = \quad Ae^{-i(\omega t + \phi)} \quad = \quad A\cos(\omega t + \phi) \; - \; i\,A\sin(\omega t + \phi)$$

$$y(t) \quad = \quad Ae^{i(\omega t)} \; + \; Be^{-i(\omega t)}$$

$$\vec{F} \quad = \quad -\,k\vec{x}$$

$$\vec{F} \quad = \quad -\,k\vec{x} \; - \; b\vec{v}$$

$$m\,\frac{d^2x}{dt^2} \quad = \quad -\,kx \; - \; bv$$

$$m\,\frac{d^2x}{dt^2} \; + \; b\frac{dx}{dt} \; + \; kx \quad = \quad 0$$

$$x(t) \quad = \quad A\cos(\omega t + \phi)\, e^{-t/\tau}$$

$$x(t) \quad = \quad Ae^{i(\omega t + \phi) - (t/\tau)}$$

$$v(t) \quad = \quad \frac{dx}{dt} \quad = \quad \left[i\omega - \frac{1}{\tau} \right] Ae^{i(\omega t + \phi) - (t/\tau)} \quad = \quad \left[i\omega - \frac{1}{\tau} \right] x(t)$$

$$a(t) \quad = \quad \frac{d^2x}{dt^2} \quad = \quad \left[i\omega - \frac{1}{\tau} \right]^2 Ae^{i(\omega t + \phi) - (t/\tau)} \quad = \quad \left([\frac{1}{\tau^2} - \omega^2] \; - \; i(\frac{2\omega}{\tau}) \right) x(t)$$

$$\left([\frac{m}{\tau^2} - m\omega^2] \; - \; i[\frac{2m\omega}{\tau}] \right) x(t) \; + \; \left(-\frac{b}{\tau} \; + i\,b\omega \right) x(t) \; + \; (k)\,x(t) \quad = \quad 0$$

$$-\frac{2m\omega}{\tau} \; + \; b\omega \quad = \quad 0$$

$$\tau = \frac{2m}{b}$$

$$\frac{m}{\tau^2} - m\omega^2 - \frac{b}{\tau} + k = 0$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$\omega = \sqrt{{\omega_0}^2 - \frac{b^2}{4m^2}}$$

$$x(t) = Ae^{i(\omega t+\phi)-(t/\infty)} = Ae^{i(\omega t+\phi)}$$

$$x(t) = Ae^{i(0+\phi)-(t/\tau)} = ACe^{-t/\tau}$$

$$x(t) = Bte^{-t/\tau}$$

$$x(t) = Ae^{-t/\tau} + Bte^{-t/\tau}$$

$$\gamma = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$$

$$\omega = \pm i\gamma$$

$$x(t) = Ae^{-(\gamma+\frac{1}{\tau})t} + Be^{-(\frac{1}{\tau}-\gamma)t}$$