$$\vec{F_{\rm air}} \quad = \quad -b\vec{v}$$

$$F_1 \quad = \quad -kx_1 \; + \; k(x_2 - x_1) \; - \; bv_1 \quad = \quad -2kx_1 \; + \; kx_2 \; - \; bv_1$$

$$F_2 \quad = \quad -k(x_2 - x_1) \; - \; kx_2 \; - \; bv_2 \quad = \quad kx_1 \; - \; 2kx_2 \; - \; bv_2$$

$$m\frac{d^2x_1}{dt^2} \quad = \quad -2kx_1 \; + \; kx_2 \; - \; b\,\frac{dx_1}{dt}$$

$$m\frac{d^2x_2}{dt^2} \quad = \quad kx_1 \; - \; 2kx_2 \; - \; b\,\frac{dx_2}{dt}$$

$$m\frac{d^2(x_1+x_2)}{dt^2} \quad = \quad -k(x_1+x_2) \; - \; b\,\frac{d(x_1+x_2)}{dt}$$

$$s_1 \quad \equiv \quad x_1 \; + \; x_2$$

$$m\frac{d^2s_1}{dt^2} \quad = \quad -ks_1 \; - \; b\frac{ds_1}{dt}$$

$$s_1(t) \quad = \quad A_1\cos(\omega_1 t + \phi_1)e^{-t/\tau}$$

$$s_1(t) \quad = \quad A_1 e^{i(\omega_1 t+\phi_1)-(t/\tau)}$$

$$\frac{ds_1}{dt} \quad = \quad \left(i\omega_1 - \frac{1}{\tau}\right) \; A_1 e^{i(\omega_1 t+\phi_1)-(t/\tau)}$$

$$\frac{d^2s_1}{dt^2} \quad = \quad \left(-\omega_1^2 - i\frac{2\omega_1}{\tau} + \frac{1}{\tau^2}\right) A_1 e^{i(\omega_1 t+\phi_1)-(t/\tau)}$$

$$m\frac{d^2s_1}{dt^2} \; + \; b\frac{ds_1}{dt} \; + \; ks_1 \quad = \quad 0$$

$$m\left(-\omega_1^2 - i\frac{2\omega_1}{\tau} + \frac{1}{\tau^2}\right) + b\left(i\omega_1 - \frac{1}{\tau}\right) + k = 0$$

$$-\frac{2\omega_1}{\tau} + b\omega_1 = 0$$

$$b = \frac{2m}{\tau} \qquad \text{or} \qquad \tau = \frac{2m}{b}$$

$$-\omega_1^2 + \frac{1}{\tau^2} - \frac{b}{m\tau} + \frac{k}{m} = 0$$

$$\omega_1^2 = \frac{k}{m} - \frac{b^2}{4m^2}$$

$$m\frac{d^2(x_2 - x_1)}{dt^2} = -3k(x_2 - x_1) - b\frac{d(x_2 - x_1)}{dt}$$

$$m\frac{d^2 s_2}{dt^2} + b\frac{ds_2}{dt} + 3ks_2 = 0$$

$$s_2(t) = A_2 e^{i(\omega_2 t + \phi_2) - (t/\tau)}$$

$$\omega_2^2 = \frac{3k}{m} - \frac{b^2}{4m^2}$$

$$\begin{aligned} s_1 &= x_1 + x_2 \\ s_2 &= x_2 - x_1 \end{aligned}$$

$$\begin{aligned} x_1 &= \frac{1}{2}(s_1 - s_2) \\ x_2 &= \frac{1}{2}(s_1 + s_2) \end{aligned}$$

$$x_1 = \frac{1}{2}\left(A_1 e^{i(\omega_1 t + \phi_1) - (t/\tau)} - A_2 e^{i(\omega_2 t + \phi_2) - (t/\tau)}\right)$$

$$x_2 = \frac{1}{2}\left(A_1 e^{i(\omega_1 t + \phi_1) - (t/\tau)} + A_2 e^{i(\omega_2 t + \phi_2) - (t/\tau)}\right)$$

$$x_1(t) \quad = \quad A_1 e^{i(\omega_1 t + \phi_1) - (t/\tau)}$$

$$\frac{dx_1}{dt} \quad = \quad \left(i\omega \; - \; \frac{1}{\tau} \right) \; x_1$$

$$\begin{aligned} \frac{d^2 x_1}{dt^2} \quad &= \quad \left(i\omega \; - \; \frac{1}{\tau} \right)^2 \; x_1 \\ &= \quad \left(-\omega^2 \; - \; i\frac{2\omega}{\tau} \; + \; \frac{1}{\tau^2} \right) \; x_1 \end{aligned}$$

$$\begin{pmatrix} -2\frac{k}{m} + \frac{b}{m\tau} - i\frac{b\omega}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} + \frac{b}{m\tau} - i\frac{b\omega}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \left(-\omega^2 - i\frac{2\omega}{\tau} + \frac{1}{\tau^2} \right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} -2\frac{k}{m} + \frac{b}{m\tau} - i\frac{b\omega}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} + \frac{b}{m\tau} - i\frac{b\omega}{m} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \left(-\omega^2 - i\frac{2\omega}{\tau} + \frac{1}{\tau^2} \right) \begin{pmatrix} p \\ q \end{pmatrix}$$

$$s_1 \quad = \quad p x_1 \; + \; q x_2$$

$$s_1 \quad = \quad 1x_1 \; + \; 1x_2 \qquad \Rightarrow \qquad s_1 \quad = \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_2 \quad = \quad -1x_1 \; + \; 1x_2 \qquad \Rightarrow \qquad s_2 \quad = \quad \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$