$$F_1 = -kx_1 + (x_2 - x_1)k_c$$

$$F_2 = -k_c(x_2 - x_1) - kx_2$$

$$-\left(\frac{k}{m} + \frac{k_c}{m}\right)x_1 + \frac{k_c}{m}x_2 = \frac{d^2x_1}{dt^2}$$

$$\frac{k_c}{m}x_1 - \left(\frac{k}{m} + \frac{k_c}{m}\right)x_2 = \frac{d^2x_2}{dt^2}$$

$$\begin{pmatrix} -\left(\frac{k}{m} + \frac{k_c}{m}\right) & \frac{k_c}{m} \\ \frac{k_c}{m} & -\left(\frac{k}{m} + \frac{k_c}{m}\right) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

$$s = ax_1 + bx_2$$

$$\begin{pmatrix} -\left(\frac{k}{m} + \frac{k_c}{m}\right) & \frac{k_c}{m} \\ \frac{k_c}{m} & -\left(\frac{k}{m} + \frac{k_c}{m}\right) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\gamma \equiv \left(\frac{k}{m} + \frac{k_c}{m}\right)$$

$$\Delta \equiv \frac{k_c}{m}$$

$$\begin{pmatrix} -\gamma & \Delta \\ \Delta & -\gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$-\gamma a + \Delta b + \omega^2 a = 0$$

$$\Delta a - \gamma b + \omega^2 b = 0$$

$$b = \frac{(\gamma - \omega^2)}{\Delta}a$$

$$\Delta\, a - \gamma\left(\frac{\gamma - \omega^2}{\Delta}\right)a + \omega^2\left(\frac{\gamma - \omega^2}{\Delta}\right)a = 0$$

$$(\Delta^2 - \gamma^2) + 2\gamma\omega^2 - \omega^4 = 0$$

$$\lambda \quad \equiv \quad \omega^2$$

$$(\Delta^2 - \gamma^2) \; + \; 2\gamma\lambda \; - \; \lambda^2 \quad = \quad 0$$

$$\lambda \quad = \quad \gamma \; \pm \; \Delta \quad = \quad \left(\frac{k}{m} + \frac{k_c}{m} \right) \; \pm \; \frac{k_c}{m}$$

$$\lambda \quad = \quad \frac{k}{m} \; + \; \frac{2k_c}{m} \qquad \text{or} \qquad \lambda \quad = \quad \frac{k}{m}$$

$$\omega_1 \quad = \quad \sqrt{\frac{k}{m}}$$

$$\omega_2 \quad = \quad \sqrt{\frac{k}{m} \; + \; \frac{2k_c}{m}}$$

$$-\gamma a \; + \; \Delta b \; + \; \left(\frac{k}{m} + \frac{2k_c}{m} \right) a \quad = \quad 0$$

$$-\left(\frac{k}{m} + \frac{k_c}{m} \right) a \; + \; \left(\frac{k_c}{m} \right) b \; + \; \left(\frac{k}{m} + \frac{2k_c}{m} \right) a \quad = \quad 0$$

$$b \quad = \quad -a$$

$$s_2 \quad = \quad \begin{pmatrix} 1x_1 \\ -1x_2 \end{pmatrix} \quad = \quad x_1 \; - \; x_2 \qquad \text{or} \qquad x_2 \; - \; x_1$$

$$b \quad = \quad a$$

$$s_1 \quad = \quad \begin{pmatrix} 1x_1 \\ 1x_2 \end{pmatrix} \quad = \quad x_1 \; + \; x_2$$

$$\begin{aligned} s_1(t) \quad &= \quad A_1 \cos(\omega_1 t + \phi_1) \\ s_2(t) \quad &= \quad A_2 \cos(\omega_2 t + \phi_2) \end{aligned}$$

$$\begin{aligned} s_1(t) \quad &= \quad A_1 \cos(\omega_1 t + \phi_1) \quad = \quad x_1(t) \; + \; x_2(t) \\ s_2(t) \quad &= \quad A_2 \cos(\omega_2 t + \phi_2) \quad = \quad x_2(t) \; - \; x_1(t) \end{aligned}$$

$$a = a$$

$$x_1(t) = \frac{1}{2}\bigg(A_1\cos(\omega_1 t + \phi_1) \ - \ A_2\cos(\omega_2 t + \phi_2)\bigg)$$

$$x_2(t) = \frac{1}{2}\bigg(A_1\cos(\omega_1 t + \phi_1) \ + \ A_2\cos(\omega_2 t + \phi_2)\bigg)$$

$$x_1(t) = \frac{L}{2}\bigg[\cos(\omega_1 t) \ - \ \cos(\omega_2 t)\bigg]$$

$$x_2(t) = \frac{L}{2}\bigg[\cos(\omega_1 t) \ + \ \cos(\omega_2 t)\bigg]$$