$$\text{matrix } \mathbf{A} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad = \quad \begin{pmatrix} 2+4 \\ 5-2 \end{pmatrix} \quad = \quad \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad = \quad \begin{pmatrix} 2+2 \\ 5-1 \end{pmatrix} \quad = \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad = \quad 4\mathbf{x}$$

$$\text{vector } \mathbf{x} \quad = \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad = \quad \begin{pmatrix} 4+4 \\ 10-2 \end{pmatrix} \quad = \quad \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{vs} \quad \mathbf{Ax} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$\mathbf{Ax} \quad = \quad \lambda\,\mathbf{x}$$

$$\begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad = \quad \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{aligned} 2x \;+\; 2y \quad &= \quad \lambda x \\ 5x \;-\; 1y \quad &= \quad \lambda y \end{aligned}$$

$$2y \quad = \quad (\lambda - 2)x \qquad \Rightarrow \qquad y \quad = \quad \frac{1}{2}(\lambda - 2)x$$

$$5x \;-\; \frac{1}{2}\lambda x \;+\; x \quad = \quad \frac{1}{2}\lambda^2 x \;-\; \lambda x$$

$$\frac{1}{2}\lambda^2 \; - \; \frac{1}{2}\lambda \; - \; 6 \quad = \quad 0$$

$$\lambda^2 \; - \; \lambda \; - \; 12 \quad = \quad 0$$

$$\lambda = 4 \quad , \quad \lambda = -3$$

$$\begin{aligned} 2x \; + \; 2y \quad &= \quad 4x \\ 5x \; - \; 1y \quad &= \quad 4y \end{aligned}$$

$$\begin{aligned} 2x \quad = \quad 2y \qquad &\Rightarrow \quad x \; = \; y \\ 5x \quad = \quad 5y \qquad &\Rightarrow \quad x \; = \; y \end{aligned}$$

$$\text{eigenvector corresponding to } \lambda = 4 \quad \rightarrow \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ a \end{pmatrix}$$

$$\text{eigenvector corresponding to } \lambda = -3 \quad \rightarrow \quad \begin{pmatrix} -2 \\ 5 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 4 \\ -10 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -2a \\ 5a \end{pmatrix}$$

$$\frac{d^2x_1}{dt^2} \; + \; 2\frac{k}{m}x_1 \; - \; \frac{k}{m}x_2 \quad = \quad 0$$

$$\frac{d^2x_2}{dt^2} \; - \; \frac{k}{m}x_1 \; + \; 2\frac{k}{m}x_2 \quad = \quad 0$$

$$-2\frac{k}{m}x_1 \; + \; \frac{k}{m}x_2 \quad = \quad \frac{d^2x_1}{dt^2}$$

$$\frac{k}{m}x_1 \; - \; 2\frac{k}{m}x_2 \quad = \quad \frac{d^2x_2}{dt^2}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad = \quad \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

$$s \quad = \quad \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix}$$

$$\frac{d^2s}{dt^2} \quad = \quad -\omega^2 s$$

$$\frac{d^2x_1}{dt^2} \quad = \quad -\omega^2 x_1$$

$$\frac{d^2x_2}{dt^2} \quad = \quad -\omega^2 x_2$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix} \quad = \quad -\omega^2 \begin{pmatrix} ax_1 \\ bx_2 \end{pmatrix}$$

$$\mathbf{As} \quad = \quad \lambda\,\mathbf{s}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad -\omega^2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad \omega^2 \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \; + \; 1(\frac{k}{m})b \; + \; \omega^2 a \quad = \quad 0$$

$$1(\frac{k}{m})a \; - \; 2(\frac{k}{m})b \; + \; \omega^2 b \quad = \quad 0$$

$$b \quad = \quad \left( \frac{\omega^2 - 2\frac{k}{m}}{-\frac{k}{m}} \right) a$$

$$1(\frac{k}{m})a \; + \; \left( -2\frac{k}{m} + \omega^2 \right) b \quad = \quad 0$$

$$1(\frac{k}{m})a \; + \; \left( -2\frac{k}{m} + \omega^2 \right) \left( \frac{\omega^2 - 2\frac{k}{m}}{-\frac{k}{m}} \right) a \quad = \quad 0$$

$$-1 \left( \frac{k}{m} \right)^2 \; + \; 4 \left( \frac{k}{m} \right)^2 \; - \; 4 \left( \frac{k}{m} \right) \omega^2 \; + \; \omega^4 \quad = \quad 0$$

$$\lambda \quad = \quad \omega^2$$

$$3 \left( \frac{k}{m} \right)^2 \; - \; 4 \left( \frac{k}{m} \right) \lambda \; + \; \lambda^2 \quad = \quad 0$$

$$\lambda \quad = \quad \omega^2 \quad = \quad 1\frac{k}{m} \quad \text{or} \quad 3\frac{k}{m}$$

$$\omega_1 \quad = \quad \sqrt{\frac{k}{m}}$$

$$\omega_2 \quad = \quad \sqrt{\frac{3k}{m}}$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad 1(\frac{k}{m}) \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \; + \; 1(\frac{k}{m})b \; + \; 1(\frac{k}{m})a \quad = \quad 0$$

$$1(\frac{k}{m})a \; - \; 2(\frac{k}{m})b \; + \; 1(\frac{k}{m})b \quad = \quad 0$$

$$a \quad = \quad b$$

$$\text{eigenvalue } \omega_1 \; = \; \sqrt{\frac{k}{m}} \quad \text{corresponds to eigenvector} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_1 \quad = \quad \begin{pmatrix} 1x_1 \\ 1x_2 \end{pmatrix} \qquad \text{or} \qquad s_1 \quad = \quad x_1 \; + \; x_2$$

$$\begin{pmatrix} -2\frac{k}{m} & \frac{k}{m} \\ \frac{k}{m} & -2\frac{k}{m} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad + \quad 3(\frac{k}{m}) \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$-2(\frac{k}{m})a \; + \; 1(\frac{k}{m})b \; + \; 3(\frac{k}{m})a \quad = \quad 0$$

$$1(\frac{k}{m})a \; - \; 2(\frac{k}{m})b \; + \; 3(\frac{k}{m})b \quad = \quad 0$$

$$a \quad = \quad -b$$

$$\text{eigenvalue } \omega_2 \; = \; \sqrt{\frac{3k}{m}} \quad \text{corresponds to eigenvector} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$s_2 \quad = \quad \begin{pmatrix} 1x_1 \\ -1x_2 \end{pmatrix} \qquad \text{or} \qquad s_2 \; = \; x_1 \; - \; x_2 \quad \text{or} \quad s_2 \; = \; x_2 \; - \; x_1$$

$$(k/m) \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad - \quad \lambda \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$(k/m) \begin{pmatrix} -2-\lambda & 1 \\ 1 & -2-\lambda \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$F_1 \quad = \quad -kx_1 \; + \; 2k(x_2 - x_1)$$

$$F_2 \quad = \quad -2k(x_2 - x_1) \; - \; 3kx_2$$

$$\begin{aligned} F_1 \quad &= \quad -3kx_1 \; + \; 2kx_2 \\ F_2 \quad &= \quad 2kx_1 \; - \; 5kx_2 \end{aligned}$$

$$\begin{aligned} m\,\frac{d^2x_1}{dt^2} \quad &= \quad -3kx_1 \; + \; 2kx_2 \\ m\,\frac{d^2x_2}{dt^2} \quad &= \quad 2kx_1 \; - \; 5kx_2 \end{aligned}$$

$$\begin{aligned} \left(\frac{k}{m}\right)\left[-3x_1 \; + \; 2x_2\right] \quad &= \quad \frac{d^2x_1}{dt^2} \\ \left(\frac{k}{m}\right)\left[2x_1 \; - \; 5x_2\right] \quad &= \quad \frac{d^2x_2}{dt^2} \end{aligned}$$

$$\left(\frac{k}{m}\right)\begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad = \quad \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

$$s \quad = \quad ax_1 \; + \; bx_2$$

$$\frac{d^2s}{dt^2} \quad = \quad -\omega^2\, s$$

$$\left(\frac{k}{m}\right)\begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad -\omega^2\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{aligned} -3a \; + \; 2b \quad &= \quad -\left(\frac{\omega^2}{k/m}\right) a \\ 2a \; - \; 5b \quad &= \quad -\left(\frac{\omega^2}{k/m}\right) b \end{aligned}$$

$$b \quad = \quad -\frac{1}{2}\frac{\omega^2}{k/m}\, a \; + \; \frac{3}{2}\, a$$

$$\frac{\omega^4}{k^2/m^2} \; + \; 8\frac{\omega^2}{k/m} \; + \; 11 \quad = \quad 0$$

$$\left(\frac{k}{m}\right)\begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad \lambda\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad \lambda\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \; - \; \lambda \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$\begin{pmatrix} -3-\lambda & 2 \\ 2 & -5-\lambda \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad 0$$

$$(-3-\lambda)(-5-\lambda) \; - \; (2)(2)$$

$$\lambda^2 \; + \; 8\lambda \; + \; 11 \quad = \quad 0$$

$$\lambda \quad = \quad -\omega^2$$

$$\begin{aligned} \omega_1 \quad &= \quad \sqrt{1.76\frac{k}{m}} \\ \omega_2 \quad &= \quad \sqrt{6.24\frac{k}{m}} \end{aligned}$$

$$\left(\frac{k}{m}\right) \begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad -1.76\frac{k}{m} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\left(\frac{k}{m}\right) \begin{pmatrix} -3 & 2 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad = \quad -6.24\frac{k}{m} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{aligned} {\omega_1}^2 = 1.76\frac{k}{m} \quad &\rightarrow \quad a \quad = \quad 1.61\, b \\ {\omega_2}^2 = 6.24\frac{k}{m} \quad &\rightarrow \quad a \quad = \quad -0.62\, b \end{aligned}$$

$$\begin{aligned} s_1 \quad &= \quad 1.61x_1 \; + \; x_2 \\ s_2 \quad &= \quad -0.62x_1 \; + \; x_2 \end{aligned}$$

$$\begin{aligned} s_1(t) \quad &= \quad A_1 \cos(\omega_1 t \; + \; \phi_1) \\ s_2(t) \quad &= \quad A_2 \cos(\omega_2 t \; + \; \phi_2) \end{aligned}$$

$$\begin{aligned} s_1(t) \quad &= \quad L \cos(\omega_1 t) \\ s_2(t) \quad &= \quad L \cos(\omega_2 t) \end{aligned}$$

$$\lambda \quad \equiv \quad -\omega^2 \, (\frac{m}{k})$$

$$\omega \quad \equiv \quad \sqrt{-\lambda \, \frac{k}{m}}$$