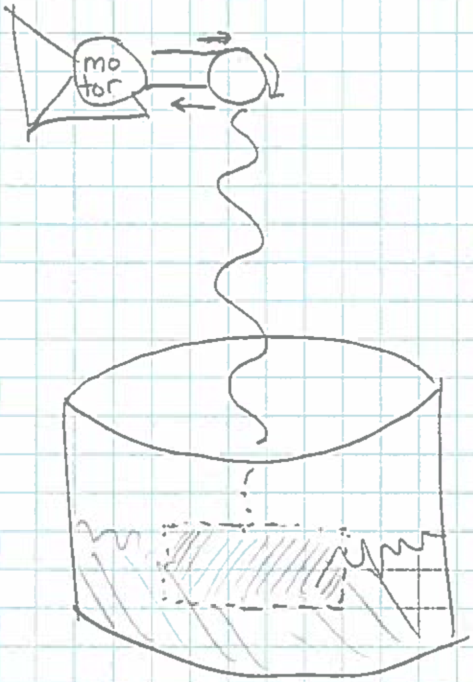
Fred must design a device to stir thick chocolate liquid in a giant vat.



He mounts a motor on the wall above the vat, and hangs a wire down into that vat.

Wire has

$$\mu = 0.053 \ \frac{kg}{m}$$

$$T = 33 \ N$$

The impedance of the wire is

$$Z = \sqrt{T\mu} = \sqrt{(33 \ N)(0.053 \ \tfrac{kg}{m})}$$

$$= 1.32 \ kg/s$$

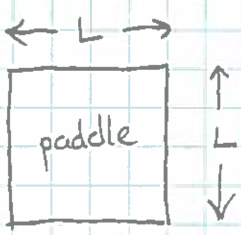
Fred can choose a paddle of any size to stir the liquid chocolate. Which size will cause the maximum energy transfer from wire to liquid?

Fred runs tests using square paddles of various sizes. He determines that the force of "chocolate resistance" on the paddle is

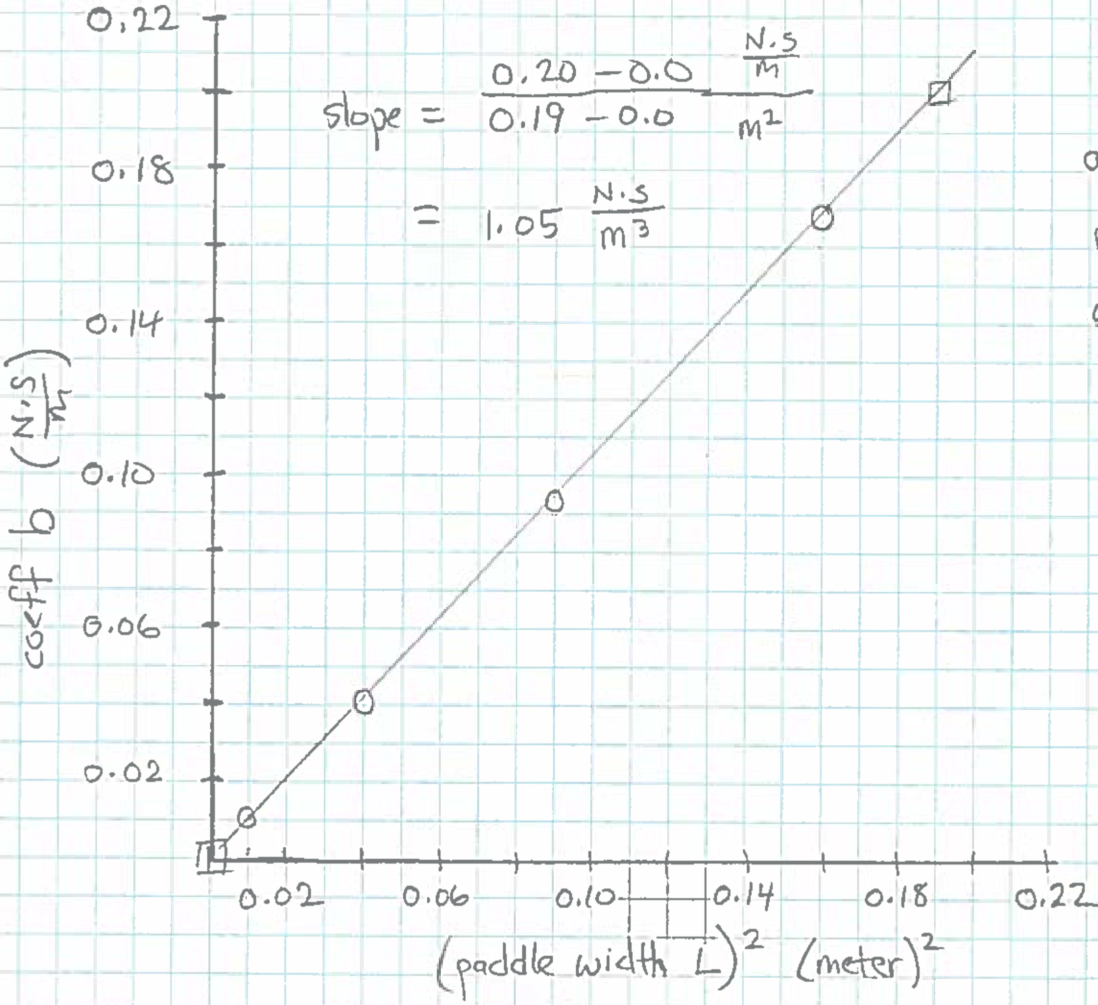
$$\vec{F} = -b\vec{v}$$

where $\vec{v}$ is velocity of paddle in m/s, and b is some coefficient.

His tests show



paddle width (m)	coeff b $\left(\frac{N \cdot s}{m}\right)$
0.10	0.010543
0.20	0.042171
0.30	0.094886
0.40	0.168686



$$\text{slope} = \frac{0.20 - 0.0 \ \frac{N \cdot s}{m}}{0.19 - 0.0 \ m^2}$$

$$= 1.05 \ \frac{N \cdot s}{m^3}$$

Fred notes that there appears to be a quadratic relationship; a graph confirms his guess

$$b = 1.05 \ \frac{N \cdot s}{m^3} \cdot L^2$$

So, he knows the size of the paddle required to yield some desired value of "b"

Which "b" is best? To match the impedance of the wire,

$$b = Z \qquad \text{(see "Impedance" lecture)}$$

$$\Rightarrow 1.05 \cdot L^2 = Z$$

$$L = \sqrt{\frac{Z}{1.05}} = \sqrt{\frac{1.32 \ kg/s}{1.05 \ \frac{N \cdot s}{m^3}}}$$

$$\boxed{L = 1.12 \ m}$$