Joan observes a wave travelling along a string. When the wave reaches a section of opaque glass, it disappears — but Joan is told that it connects to a different type of string, kept at same tension.

<u>Visible region</u>

Tension $T = 50$ N

ang freq $\omega = 5$ rad/s

linear dens $\mu_I = 0.009$ kg/m

amplitude $A_i = 1.0$ m

We can compute

wave speed $V_I = \sqrt{\frac{T}{\mu_I}} = 74.54$ m/s

and thus

$$\text{Power } P_I = \frac{1}{2}(\mu_I V_I)\,\omega^2 A_i^2$$
$$= 8.38 \text{ W}$$

After the incident wave reaches the invisible boundary, some of it reflects and travels back to the visible section, interfering with the incident wave. Joan sees a combined amplitude

$$A_i + A_r = 1.3 \text{ m} \Rightarrow A_r = 0.3 \text{ m}$$

What is the linear mass density μ_{II} on the other side of the boundary?

First, note that the linear density and wave number are related

$$k_I = \omega\sqrt{\frac{\mu_I}{T}} \qquad k_{II} = \omega\sqrt{\frac{\mu_{II}}{T}}$$

Since ω is same on both sides, and tension T is also the same, we have

$$k \propto \sqrt{\mu}$$

So we can replace all the wave numbers k in

$$A_r = \frac{k_{II} - k_I}{k_I + k_{II}} A_i$$

with $\sqrt{\mu}$

$$A_r = \frac{\sqrt{\mu_{II}} - \sqrt{\mu_I}}{\sqrt{\mu_I} + \sqrt{\mu_{II}}} A_i$$

Solving for μ_{II}, we have

$$A_r \sqrt{\mu_I} + A_r \sqrt{\mu_{II}} = A_i \sqrt{\mu_{II}} - A_i \sqrt{\mu_I}$$

$$\sqrt{\mu_{II}} (A_i - A_r) = \sqrt{\mu_I} (A_i + A_r)$$

$$\mu_{II} = \mu_I \left(\frac{A_i + A_r}{A_i - A_r} \right)^2 = \left(0.009 \frac{kg}{m}\right) \left(\frac{1.3}{0.7} \right)^2$$

$$= 3.10 \times 10^{-2} \ kg/m$$

Now we can compute the power transmitted into the invisible region

$$P_{II} = \frac{1}{2} (\mu_{II} v_{II}) \omega^2 A_t^2$$

Here $\quad v_{II} = \sqrt{\frac{T}{\mu_{II}}} = 40.16 \frac{m}{s} \qquad A_t = \left(\frac{2\sqrt{\mu_I}}{\sqrt{\mu_I} + \sqrt{\mu_{II}}} \right) A_i = 0.7m$

So

$$P_{II} = \frac{1}{2} \left(3.10 \times 10^{-2} \frac{kg}{m}\right) \left(40.16 \frac{m}{s}\right) \left(5 \frac{rad}{s}\right)^2 (0.7m)^2$$

$$= 7.63 \ W$$