

← L →



A piece of string of length  $L$ , mass  $M$  is stretched between two posts. It suffers uniform tension  $T$  all along its length.

What are the three lowest angular frequencies of vibration? In other words, what are the angular frequencies of the three lowest normal modes:  $n=1$ ,  $n=2$ ,  $n=3$ .

The shapes of the string in these modes are



The speed of waves on this string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{M/L}} = \sqrt{\frac{TL}{M}}$$

And since the speed of a wave is

$$v = \lambda f = \lambda \frac{\omega}{2\pi} \rightarrow$$

we can determine the angular frequency of each mode:

$$n=1 \quad \omega_1 = \frac{2\pi v}{\lambda_1} = \frac{2\pi \sqrt{\frac{TL}{M}}}{2L} = \pi \sqrt{\frac{T}{ML}}$$

$$n=2 \quad \omega_2 = \frac{2\pi v}{\lambda_2} = \frac{2\pi \sqrt{\frac{TL}{M}}}{L} = 2\pi \sqrt{\frac{T}{ML}}$$

$$n=3 \quad \omega_3 = \frac{2\pi v}{\lambda_3} = \frac{2\pi \sqrt{\frac{TL}{M}}}{\frac{2}{3}L} = 3\pi \sqrt{\frac{T}{ML}}$$