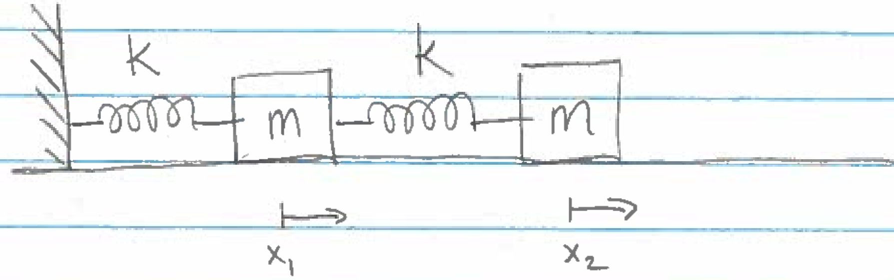

Two blocks, of mass m, slide on a frictionless floor. They are connected by two identical springs of force constant k.

The forces on the blocks are

$$m\frac{d^2x_1}{dt^2} = -kx_1 + k(x_2 - x_1) = -2kx_1 + kx_2$$

$$m\frac{d^2x_2}{dt^2} = -k(x_2 - x_1) = +kx_1 - kx_2$$

which can be converted to matrix form

$$\left(\frac{k}{m}\right)\begin{bmatrix} -2 & +1 \\ +1 & -1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

If we assume some linear combination of positions

$$s = ax_1 + bx_2$$

will cause the blocks to exhibit SHM with the same frequency ω, then

$$\left(\frac{k}{m}\right)\begin{bmatrix} -2 & +1 \\ +1 & -1 \end{bmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2\begin{pmatrix} a \\ b \end{pmatrix}$$

We look for eigenvalues of this matrix in the usual way.
From top row,

$$-2\frac{k}{m}a + \frac{k}{m}b = -\omega^2 a$$

$$\rightarrow \qquad b = \left(\frac{2\frac{k}{m} - \omega^2}{\frac{k}{m}}\right) a$$

Then substitute this into the second row

$$\frac{k}{m}a - 1\frac{k}{m}\left(\frac{2\frac{k}{m} - \omega^2}{\frac{k}{m}}\right) a = -\omega^2\left(\frac{2\frac{k}{m} - \omega^2}{\frac{k}{m}}\right) a$$

$$\left(\frac{k}{m}\right)^2 - 2\left(\frac{k}{m}\right)^2 + \frac{k}{m}\omega^2 = -2\frac{k}{m}\omega^2 + \omega^4$$

$$\omega^4 - 3\frac{k}{m}\omega^2 + 1\left(\frac{k}{m}\right)^2 = 0$$

Let $\lambda = \omega^2$. Then

$$\lambda^2 - 3\frac{k}{m}\lambda + 1\left(\frac{k}{m}\right)^2 = 0$$

$$\lambda = \frac{3\frac{k}{m} \pm \sqrt{9\frac{k^2}{m^2} - 4\frac{k^2}{m^2}}}{2}$$

$$\lambda = \frac{3\frac{k}{m} \pm \sqrt{5}\frac{k}{m}}{2} = \begin{cases} \frac{3+\sqrt{5}}{2}\frac{k}{m} \\ \frac{3-\sqrt{5}}{2}\frac{k}{m} \end{cases}$$

Therefore, the frequencies of the normal modes are

$$\boxed{\begin{aligned} \omega_2 &= \sqrt{\frac{3+\sqrt{5}}{2}\frac{k}{m}} \\ \omega_1 &= \sqrt{\frac{3-\sqrt{5}}{2}\frac{k}{m}} \end{aligned}}$$

higher freq

lower freq

To find the ratio of the amplitudes of each block's motion, we need the eigenvectors corresponding to each frequency.

For

higher freq mode $\longrightarrow$ $\omega_2 = \sqrt{\frac{3+\sqrt{5}}{2}\frac{k}{m}}$ Plug into equation from top row of the force matrix

$$-2\frac{k}{m}a + \frac{k}{m}b = -\omega_2^2 a$$

$$-2\frac{k}{m}a + \frac{k}{m}b = -2.618\frac{k}{m}a$$

$$\rightarrow \frac{k}{m}b = -0.618\frac{k}{m}a$$

$$\rightarrow b = -0.618\,a$$

$$S_2 = 1x_1 - 0.618x_2$$

Ratio of amplitudes is $\frac{1}{-0.618} = \boxed{-1.618}$

For the lower frequency mode ω_1

$$\omega_1 = \sqrt{\frac{3-\sqrt{5}}{2}\frac{k}{m}}$$

Plugging in, we get

$$-2\frac{k}{m}a + \frac{k}{m}b = -\omega_1^2 a$$

$$-2\frac{k}{m}a + \frac{k}{m}b = -0.382\,a$$

$$\frac{k}{m}b = +1.618\,a$$

$$\rightarrow \quad b = 1.618\,a$$

$$S_1 = 1\,x_1 + 1.618\,x_2$$

Ratio of amplitudes is

$$\frac{1}{1.618} = \boxed{0.618}$$