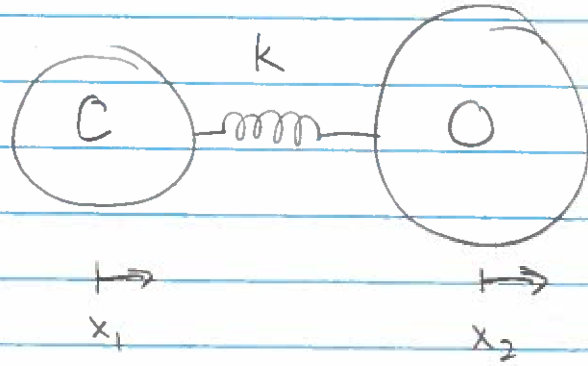

Carbon monoxide consists of ordinary ^{12}C and ^{16}O atoms, which have masses

$$^{12}C = 12 \times m_H = 12 \times 1.67 \times 10^{-27}\ kg$$
$$^{16}O = 16 \times m_H = 16 \times 1.67 \times 10^{-27}\ kg$$

So

$$m_C = \frac{12}{16} m_o = \frac{3}{4} m_o$$

We can write the sum of forces on each atom as

$$m_C \frac{d^2x_1}{dt^2} = \frac{3}{4} m_o \frac{d^2x_1}{dt^2} = -kx_1 + kx_2$$

$$m_o \frac{d^2x_2}{dt^2} = 1 m_o \frac{d^2x_2}{dt^2} = +kx_1 - kx_2$$

where k = force constant of the bond between atoms.

We can re-write as a matrix equation

$$\frac{d^2x_1}{dt^2} = -\frac{4}{3}\frac{k}{m_o} x_1 + \frac{4}{3}\frac{k}{m_o} x_2$$

$$\frac{d^2x_2}{dt^2} = +\frac{k}{m_o} x_1 - \frac{k}{m_o} x_2$$

Or

$$\left(\frac{k}{m_o}\right)\begin{bmatrix} -\frac{4}{3} & +\frac{4}{3} \\ +1 & -1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

Now, if we assume that some linear combination of positions will cause this system to exhibit simple harmonic motion, we can write

$$S = ax_1 + bx_2$$

and

$$\left(\frac{k}{m_o}\right)\begin{bmatrix} -\frac{4}{3} & +\frac{4}{3} \\ +1 & -1 \end{bmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\omega^2\begin{pmatrix} a \\ b \end{pmatrix}$$

To find eigenvalues, start w/ top row of matrix

$$-\frac{4}{3}\frac{k}{m_o}a + \frac{4}{3}\frac{k}{m_o}b = -\omega^2 a$$

$$\frac{4}{3}\frac{k}{m_o}b = \left(\frac{4}{3}\frac{k}{m_o} - \omega^2\right)a$$

$$\rightarrow \quad b = \left[\frac{\frac{4}{3}\frac{k}{m_o} - \omega^2}{\frac{4}{3}\frac{k}{m_o}}\right]a$$

And substitute this expression into the lower row of matrix:

$$\frac{k}{m_o}a - \frac{k}{m_o}\left[\frac{\frac{4}{3}\frac{k}{m_o} - \omega^2}{\frac{4}{3}\frac{k}{m_o}}\right]a = -\omega^2 a\left[\frac{\frac{4}{3}\frac{k}{m_o} - \omega^2}{\frac{4}{3}\frac{k}{m_o}}\right]$$

Simplify and try to solve for the frequency ω^2:

$$\frac{4}{3}\left(\frac{k}{m_o}\right)^2 - \frac{4}{3}\left(\frac{k}{m_o}\right)^2 + \frac{k}{m_o}\omega^2 = -\frac{4}{3}\frac{k}{m_o}\omega^2 + \omega^4.$$

$$\rightarrow \quad \frac{7}{3}\frac{k}{m_o}\omega^2 - \omega^4 = 0$$

$$\rightarrow \quad \omega^2 = \frac{7}{3}\frac{k}{m_o}$$

So the frequency of this vibration should be

$$\omega = \sqrt{\frac{\frac{7}{3}(1860\ \text{N/m})}{16 \times 1.67 \times 10^{-27}\ \text{kg}}}$$

$$\omega = 4.03 \times 10^{14}\ \frac{\text{rad}}{\text{s}}$$

The wavelength will be

$$\boxed{\lambda = \frac{c}{\omega/2\pi} = \frac{2\pi c}{\omega} = 4.67 \times 10^{-6}\ \text{m}}$$

Now, if the carbon atom is ^{13}C instead of ^{12}C, then the ratio of masses is

$$m_c = \frac{13}{16} m_o$$

and the matrix of forces looks like

$$\left(\frac{K}{m_o}\right)\begin{bmatrix} -\frac{16}{13} & +\frac{16}{13} \\ +1 & -1 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{d^2x_1}{dt^2} \\ \frac{d^2x_2}{dt^2} \end{pmatrix}$$

If one performs exactly the same analysis, one ends up with

$$\omega^2 = \frac{29}{13} \frac{k}{m_0} \qquad \left(\text{instead of } \frac{7}{3} \frac{k}{m_0}\right)$$

which means for ^{13}CO

$$\omega = 3.94 \times 10^{14} \ \frac{\text{rad}}{s}$$

and

$$\boxed{\lambda = \frac{c}{\omega/2\pi} = 4.78 \times 10^{-6}\ m}$$