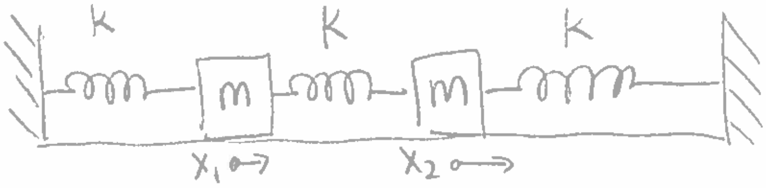

Two blocks of mass $m = 3$ kg slide on frictionless floor, attached by springs of force constant $k = 6$ N/m.

Initial conditions at $t=0$

$$x_1 = -0.8\text{ m}$$

$$x_2 = +1.6\text{ m}$$

$$v_1 = 0.8\ \tfrac{\text{m}}{\text{s}}$$

$$v_2 = 0\ \tfrac{\text{m}}{\text{s}}$$

Fred defines normal coordinates

$$s_1 = x_1 + x_2$$

$$s_2 = x_2 - x_1$$

At time $t=0$

$$s_1 = -0.8\text{ m} + 1.6\text{ m} = +0.8\text{ m}$$

$$s_2 = 1.6\text{ m} - (-0.8\text{ m}) = +2.4\text{ m}$$

$$\frac{ds_1}{dt} = v_1 + v_2 = 0.8\ \tfrac{\text{m}}{\text{s}} + 0\ \tfrac{\text{m}}{\text{s}} = 0.8\ \tfrac{\text{m}}{\text{s}}$$

$$\frac{ds_2}{dt} = v_2 - v_1 = 0\ \tfrac{\text{m}}{\text{s}} - 0.8\ \tfrac{\text{m}}{\text{s}} = -0.8\ \tfrac{\text{m}}{\text{s}}$$

The motion of the blocks, expressed in these normal coordinates, looks like SHM.

$$s_1(t) = A_1 \cos(\omega_1 t + \phi_1)$$

$$s_2(t) = A_2 \cos(\omega_2 t + \phi_2)$$

What are the frequencies ω_1 and ω_2? We can find them by writing down the forces on each block

$$F_1 = -kx_1 + k(x_2 - x_1) = -2kx_1 + kx_2 = m\frac{d^2x_1}{dt^2}$$

$$F_2 = -kx_2 - k(x_2 - x_1) = kx_1 - 2kx_2 = m\frac{d^2x_2}{dt^2}$$

Re-arranging

$$\frac{d^2x_1}{dt^2} + 2\frac{k}{m}x_1 - \frac{k}{m}x_2 = 0$$

$$\frac{d^2x_2}{dt^2} - \frac{k}{m}x_1 + 2\frac{k}{m}x_2 = 0$$

If we add these two equations, we find

$$\frac{d^2(x_1 + x_2)}{dt^2} + \frac{k}{m}(x_1 + x_2) = 0$$

$$\rightarrow \quad \frac{d^2 s_1}{dt^2} + \frac{k}{m}s = 0$$

But this is simple harmonic motion with ang frequency

$$\frac{d^2 s_1}{dt^2} = -\frac{k}{m}s_1 = -\omega_1^2 s_1 \Rightarrow \boxed{\omega_1 = \sqrt{\frac{k}{m}}} = 1.41\ \frac{\text{rad}}{\text{s}}$$

And if we <u>subtract</u> the two equations, we get

$$\frac{d^2(x_2-x_1)}{dt^2} + \frac{3k}{m}(x_2-x_1) = 0$$

$$\rightarrow \frac{d^2 s_2}{dt^2} + \frac{3k}{m} s_2 = 0$$

$$\rightarrow \boxed{\omega_2 = \sqrt{\frac{3k}{m}}} = 2.45 \,\frac{\text{rad}}{s}$$

Fine. Now we know ω_1 and ω_2, so the unknown quantities are A_1, ϕ_1 A_2, ϕ_2. We can use initial conditions

$$s_1(t=0) = A_1\cos(0+\phi_1) = A_1\cos\phi_1 = -0.8\,m$$

$$\frac{ds_1}{dt}(t=0) = -\omega_1 A_1 \sin(0+\phi_1) = -\omega_1 A_1 \sin\phi_1 = 0.8\,\frac{m}{s}$$

$$A_1 = \frac{-0.8\,m}{\cos\phi_1}$$

$$\rightarrow \omega_1\left[\frac{-0.8\,m}{\cos\phi_1}\right]\sin\phi_1 = 0.8\,\frac{m}{s}$$

$$\tan\phi_1 = \frac{0.8\,m/s}{-0.8\,m \cdot 1.41\,\frac{\text{rad}}{s}} = -0.707$$

$$\boxed{\phi_1 = -0.616\text{ rad}}$$

$$A_1 = \frac{-0.8\,m}{\cos(-0.616\text{ rad})} = \boxed{0.98\,m = A_1}$$

In the same way, we can compute the coefficients of S_2.

$$S_2(t) = A_2 \cos(\omega_2 t + \phi_2)$$

And $$S_2(t=0) = A_2 \cos(\phi_2) = 2.4\,m$$

$$\frac{dS_2}{dt}(t=0) = -\omega_2 A_2 \sin(\phi_2) = -0.8\,\frac{m}{s}$$

$$A_2 \cos\phi_2 = 2.4\,m$$

$$\Rightarrow A_2 = \frac{2.4\,m}{\cos\phi_2}$$

$$-\omega_2 \left[\frac{2.4\,m}{\cos\phi_2}\right] \sin\phi_2 = -0.8\,\frac{m}{s}$$

$$\tan\phi_2 = \frac{-0.8\,\frac{m}{s}}{-2.4\,m \cdot 2.45\,rad/s} = +0.136$$

$$\boxed{\phi_2 = +0.1352\ rad}$$

and

$$\boxed{A_2 = \frac{2.4\,m}{\cos(+0.1352\ rad)} = 2.42\ m}$$