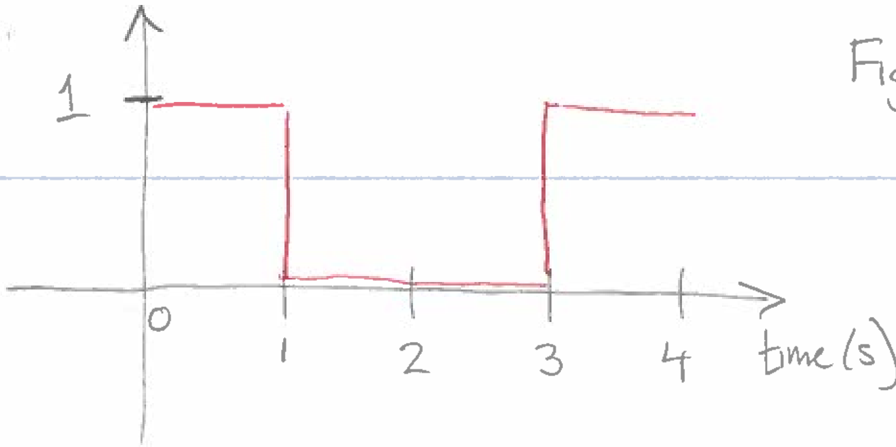

Figure at left shows one cycle of a square wave, with period $P = 4\,s$.

Find Fourier components.

$A_0 =$ averge value over one period $= \frac{1}{2}$

$B_0 = 0$ (always)

$$A_1 = \frac{2}{P}\int_0^P f(t)\cos\left(\frac{2\pi \cdot 1}{P}\cdot t\right)dt = \frac{2}{4}\int_0^1 \cos\left(\frac{\pi}{2}t\right)dt + \frac{2}{4}\int_3^4 \cos\left(\frac{\pi}{2}t\right)dt$$

$$= \frac{1}{2}\,\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right|_0^1 + \frac{1}{2}\left[\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right|_3^4\right.$$

$$= \frac{1}{\pi}\left[\sin\frac{\pi}{2} - \sin 0\right] + \frac{1}{\pi}\left[\sin(2\pi) - \sin\left(\frac{3\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}(1-0) + \frac{1}{\pi}(0-(-1)) = \boxed{\frac{2}{\pi}}$$

$$B_1 = \frac{2}{P}\int_0^P f(t)\sin\left(\frac{2\pi \cdot 1}{P}t\right)dt = \frac{2}{4}\int_0^1 \sin\left(\frac{\pi}{2}t\right)dt + \frac{2}{4}\int_3^4 \sin\left(\frac{\pi}{2}t\right)dt$$

$$= \frac{1}{2}\left[-\frac{2}{\pi}\cos\left(\frac{\pi}{2}t\right|_0^1\right. + \frac{1}{2}\left[-\frac{2}{\pi}\cos\left(\frac{\pi}{2}t\right)\right|_3^4$$

$$= -\frac{1}{\pi}\left(\cos\frac{\pi}{2} - \cos 0\right) - \frac{1}{\pi}\left[\cos(2\pi) - \cos\left(\frac{3\pi}{2}\right)\right]$$

$$= -\frac{1}{\pi}(0-1) - \frac{1}{\pi}[1-0] = \boxed{0}$$

$$A_2 = \frac{2}{P}\int_0^P f(t)\cos\left(\frac{2\pi\cdot 2}{P}\cdot t\right)dt = \frac{2}{4}\int_0^1 \cos(\pi t)dt + \frac{2}{4}\int_3^4 \cos(\pi t)dt$$

$$= \frac{1}{2}\left(\frac{1}{\pi}\right)(\sin \pi t \Big|_0^1 + \frac{1}{2}\left(\frac{1}{\pi}\right)(\sin(\pi t)\Big|_3^4$$

$$= \frac{1}{2\pi}(0-0) + \frac{1}{2\pi}(0-0) = \boxed{0}$$

$$B_2 = \frac{2}{P}\int_0^P f(t)\sin\left(\frac{2\pi\cdot 2}{P}t\right)dt = \frac{2}{4}\int_0^1 \sin(\pi t)dt + \frac{2}{4}\int_3^4 \sin(\pi t)dt$$

$$= \frac{1}{2}\left(\frac{1}{\pi}\right)[-\cos(\pi t\Big|_0^1 + \frac{1}{2}\left(\frac{1}{\pi}\right)[-\cos(\pi t\Big|_3^4$$

$$= \frac{1}{2\pi}[-(-1)-(-1)] + \frac{1}{2\pi}[-(1)-(-(-1))] = \boxed{0}$$

$$A_3 = \frac{2}{P}\int_0^P f(t)\cos\left(\frac{2\pi\cdot 3}{P}t\right)dt = \frac{2}{4}\int_0^1 \cos\left(\frac{3\pi}{2}t\right)dt + \frac{2}{4}\int_3^4 \cos\left(\frac{3\pi}{2}t\right)dt$$

$$= \frac{1}{2}\left(\frac{2}{3\pi}\right)[\sin(\frac{3\pi}{2}t\Big|_0^1 + \frac{1}{2}\left(\frac{2}{3\pi}\right)[\sin(\frac{3\pi}{2}t\Big|_3^4$$

$$= \frac{1}{3\pi}[-1-(0)] + \frac{1}{3\pi}(0-(1)) = \boxed{-\frac{2}{3\pi}}$$

$$B_3 = \frac{2}{P}\int_0^P f(t)\sin\left(\frac{2\pi\cdot 3}{P}t\right)dt = \frac{2}{4}\int_0^1 \sin\left(\frac{3\pi}{2}t\right)dt + \frac{2}{4}\int_3^4 \sin\left(\frac{3\pi}{2}t\right)dt$$

$$= \frac{1}{2}\left(\frac{2}{3\pi}\right)[-\cos(\frac{3\pi}{2}t\Big|_0^1 + \frac{1}{2}\left(\frac{2}{3\pi}\right)[-\cos(\frac{3\pi}{2}t\Big|_3^4$$

$$= \frac{1}{3\pi}[-0-(-1)] + \frac{1}{3\pi}[-1-(-0)) = \boxed{0}$$

$$A_4 = \frac{2}{P}\int_0^P f(t)\cos\left(\frac{2\pi\cdot 4}{P}t\right)dt = \frac{2}{4}\int_0^1 \cos(2\pi t)dt + \frac{2}{4}\int_3^4 \cos(2\pi t)dt$$

$$\frac{1}{2}\left(\frac{1}{2\pi}\right)\left[\sin(2\pi t\Big|_0^1 + \frac{1}{2}\left(\frac{1}{2\pi}\right)\left[\sin(2\pi t\Big|_3^4\right.\right.$$

$$= \frac{1}{4\pi}\left[0-0\right] + \frac{1}{4\pi}\left[0-0\right] = \boxed{0}$$

$$B_4 = \frac{2}{P}\int_0^P f(t)\sin\left(\frac{2\pi\cdot 4}{P}t\right)dt = \frac{2}{4}\int_0^1 \sin(2\pi t)dt + \frac{2}{4}\int_3^4 \sin(2\pi t)dt$$

$$= \frac{1}{2}\left(\frac{1}{2\pi}\right)\left[-\cos(2\pi t\Big|_0^1 + \frac{1}{2}\left(\frac{1}{2\pi}\right)\left[-\cos(2\pi t\Big|_3^4\right.\right.$$

$$= \frac{1}{4\pi}\left[-1-(-(1)\right] + \frac{1}{4\pi}\left[-1-(-(1)\right] = \boxed{0}$$