



A "skinny" square wave has a period $P=3$ s. Only the time from $t=0$ to $t=1$ will contribute to the Fourier components.

$$A_0 = \text{avg value over one period} = \boxed{\frac{1}{3}}$$

$$B_0 = 0 \quad (\text{always})$$

$$\begin{aligned} A_1 &= \frac{2}{P} \int_0^P f(t) \cos\left(\frac{2\pi \cdot 1}{P} t\right) dt = \frac{2}{3} \int_0^1 \cos\left(\frac{2\pi}{3} t\right) dt \\ &= \frac{2}{3} \left(\frac{3}{2\pi}\right) \left[\sin\left(\frac{2\pi}{3}\right) - \sin(0)\right] = \boxed{\frac{1}{\pi} \left[\sin\left(\frac{2\pi}{3}\right)\right]} \end{aligned}$$

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$$\begin{aligned} A_2 &= \frac{2}{P} \int_0^P f(t) \cos\left(\frac{2\pi \cdot 2}{P} t\right) dt = \frac{2}{3} \int_0^1 \cos\left(\frac{4\pi}{3} t\right) dt \\ &= \frac{2}{3} \left(\frac{3}{4\pi}\right) \left[\sin\left(\frac{4\pi}{3} t\right) \Big|_0^1\right] = \boxed{\frac{1}{2\pi} \left[\sin\left(\frac{4\pi}{3}\right)\right]} \end{aligned}$$

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$$A_3 = \frac{2}{P} \int_0^P f(t) \cos\left(\frac{2\pi \cdot 3}{P} t\right) dt = \frac{2}{3} \int_0^1 \cos(2\pi t) dt$$

$$= \frac{2}{3} \left(\frac{1}{2\pi}\right) [\sin(2\pi) - \sin(0)] = \boxed{0}$$

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$$A_4 = \frac{2}{P} \int_0^P f(t) \cos\left(\frac{2\pi \cdot 4}{P} t\right) dt = \frac{2}{3} \int_0^1 \cos\left(\frac{8\pi}{3} t\right) dt$$

$$= \frac{2}{3} \left(\frac{3}{8\pi}\right) [\sin\left(\frac{8\pi}{3}\right) - \sin(0)] = \boxed{\frac{1}{4\pi} \sin\left(\frac{8\pi}{3}\right)}$$

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In general

$$A_n = \begin{cases} \frac{1}{n\pi} \sin\left(\frac{2\pi n}{3}\right) & n \text{ not divis by } 3 \\ 0 & n \text{ divis by } 3 \end{cases}$$

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