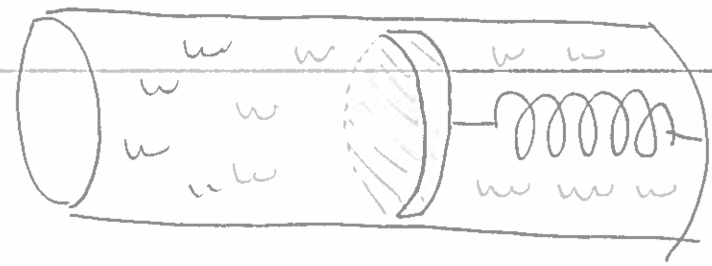

A cylinder filled with water contains a piston of mass m and spring of force constant k. The water yields a resistive force coefficient b.

The undamped frequency of the system would be

$$\omega_0 = \sqrt{\frac{k}{m}}$$

But the actual damped system oscillates at

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

The time constant of the system is

$$\tau = \frac{b}{2m}$$

Now, the piston is pulled a distance H away from its equilibrium position and held there. At $t=0$, it is released and begins to return to equilibrium. We can describe position as

$$y(t) = A\cos(\omega t + \phi)\, e^{-t/\tau}$$

And the velocity of the piston is

$$V_y(t) = \frac{dy}{dt} = -\omega A \sin(\omega t + \phi)\, e^{-t/\tau} - \frac{1}{\tau} A \cos(\omega t + \phi)\, e^{-t/\tau}$$

If we know the initial conditions

$$y(t=0) = H$$

$$v_y(t=0) = 0$$

then we can find the two parameters A (amplitude) and ϕ (phase offset angle). Just plug in $t=0$.

$$(1) \quad y(t=0) = H = A\cos(\phi)$$

$$(2) \quad V_y(t=0) = 0 = -\omega A \sin(\phi) - \frac{1}{\tau} A \cos(\phi)$$

From eqn (2)

$$\omega A \sin\phi = -\frac{1}{\tau} A \cos\phi$$

$$\frac{\sin\phi}{\cos\phi} = -\frac{\frac{1}{\tau}}{\omega}$$

$$\rightarrow \boxed{\tan\phi = -\frac{1}{\tau\omega}}$$

Then solve for A:

$$\boxed{A = \frac{H}{\cos\phi}}$$