A piston of mass m attached to spring of force constant k sits in a cylinder filled with water. Water resistance produces a resistive force with coefficient b. In this case, the resistance has been adjusted to create an <u>over-damped</u> system.

As before

$$\text{\underline{un}damped frequency } \omega_0 = \sqrt{\frac{k}{m}}$$

$$\text{time constant } \tau = \frac{b}{2m}$$

Because this is an <u>over</u>-damped system, it does not oscillate — it simply decays to equilibrium. The pseudo-frequency of the decay is

$$\gamma = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$$

The equation of motion will be

$$y(t) = A e^{-(\gamma + \frac{1}{\tau})t} + B e^{-(\frac{1}{\tau} - \gamma)t}$$

The velocity at any time is thus

$$\frac{dy}{dt} = V_y(t) = -\left(\gamma + \frac{1}{\tau}\right) A e^{-(\gamma + \frac{1}{\tau})t} - \left(\frac{1}{\tau} - \gamma\right) B e^{-(\frac{1}{\tau} - \gamma)t}$$

If we are given initial conditions at time $t=0$

$$y(t=0) = H$$

$$v_y(t=0) = 0$$

then we can find the values of parameters A and B like so:

(1) $$y(t=0) = H = A + B$$

(2) $$v_y(t=0) = 0 = -\left(\gamma + \frac{1}{\tau}\right)A - \left(\frac{1}{\tau} - \gamma\right)B$$

$\Rightarrow$ from (2)

$$A\left(\gamma + \frac{1}{\tau}\right) = -\left(\frac{1}{\tau} - \gamma\right)B$$

$$A = -\frac{\left(\frac{1}{\tau} - \gamma\right)}{\left(\gamma + \frac{1}{\tau}\right)}B$$

Plug into (1)

$$H = -\frac{\left(\frac{1}{\tau} - \gamma\right)}{\left(\gamma + \frac{1}{\tau}\right)}B + B$$

$$= B\left(1 - \frac{\frac{1}{\tau} - \gamma}{\gamma + \frac{1}{\tau}}\right)$$

$$= B\left(\frac{\gamma + \frac{1}{\tau}}{\gamma + \frac{1}{\tau}} - \frac{\frac{1}{\tau} - \gamma}{\gamma + \frac{1}{\tau}}\right)$$

$$H = B\left(\frac{2\gamma}{\gamma + \frac{1}{\tau}}\right)$$

From which

$$\boxed{B = \frac{H}{\left(\frac{2\gamma}{\gamma + \frac{1}{\tau}}\right)}}$$

and then one can compute A:

$$\boxed{A = H - B}$$