Alice designs a shock absorber for bicycle seats. It has ~~critical damping, to ensure that the rider returns to the standard~~ position as quickly as possible.

So, given a design mass $m = 58 \text{ kg}$

spring constant $k = 794 \text{ N/m}$

Alice chooses the liquid in the cylinder to yield the critical value of resistive force coeff b:

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = 0$$

$$\rightarrow \frac{b^2}{4m^2} = \frac{k}{m} \rightarrow \boxed{b = \sqrt{4km}} = 429 \frac{\text{kg}}{\text{s}}$$

The typical pothole will displace the seat to an initial position $y(t=0) = H = 0.14 \text{ m}$. How long will it take for the seat to move from this initial position, to a displacement of only 0.003 m?

This is a critically damped system, so its position as a function of time follows

$$y(t) = Ae^{-t/\tau} + Bte^{-t/\tau}$$

where

$$\tau = \frac{2m}{b} = 0.270 \text{ s}$$

We can use the initial conditions to figure out coefficients A & B.

~~$y(t=0) = H = 0.140$ m~~

$v_y(t=0) = 0$ m/s

But $\quad v_y(t) = \frac{dy}{dt} = -\frac{1}{\tau} A e^{-t/\tau} + B e^{-t/\tau} - \frac{1}{\tau} B t e^{-t/\tau}$

At time $t=0$

$$y(0) = \boxed{H = A} \rightarrow \boxed{A = 0.140 \text{ m}}$$

$$v_y(0) = 0 = -\frac{1}{\tau} A + B - 0$$

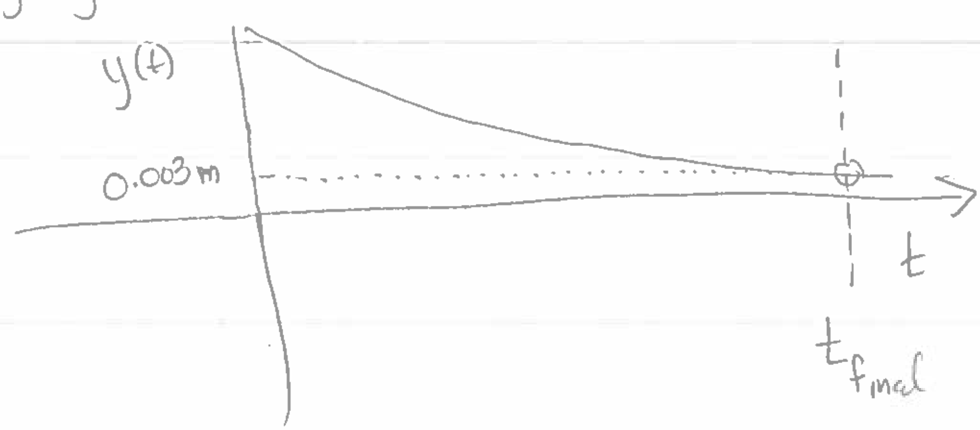
$$\rightarrow \boxed{B = \frac{1}{\tau} A} \rightarrow \boxed{B = 0.518 \tfrac{\text{m}}{\text{s}}}$$

So, now we know the position at any time:

$$y(t) = A e^{-t/\tau} + B t e^{-t/\tau}$$

$$y(t) = (0.140 \text{ m}) e^{-t/0.270\text{s}} + \left(0.518 \tfrac{\text{m}}{\text{s}}\right) t \, e^{-t/0.270\text{s}}$$

We can simply compute (or plot) $y(t)$ starting at $t=0$ and going forward in time until the position reaches 0.003 m.



Solve numerically