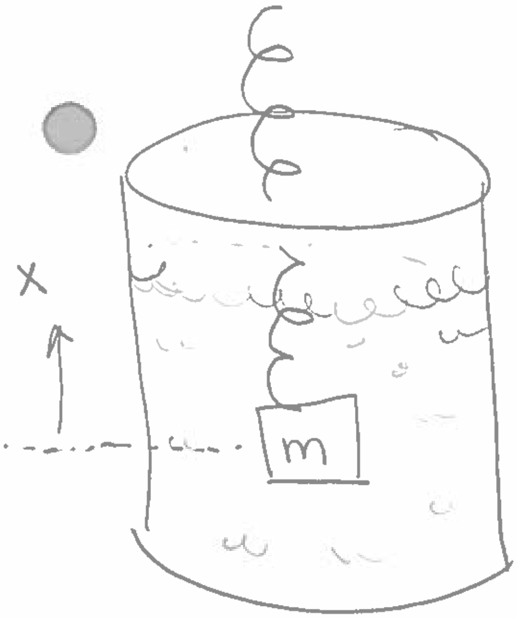

Fred hangs a mass $m = 0.98$ kg from a spring of unknown force constant k. The mass is submerged in a tank of water, and suffers resistance with coeff $b = 29.20 \frac{kg}{s}$.

When Fred pulls the mass away from equilibrium, it moves so that vertical position away from equilibrium is

$$x(t) = A e^{-29.606\, t}$$

This is a simple exponential function, not oscillatory, telling us that the system is <u>over</u>damped. Thus, the general solution is

$$x(t) = \underline{\underline{B e^{-(\gamma + \frac{1}{\tau})t}}} + C e^{-(\frac{1}{\tau} - \gamma)t}$$

Since the observed function decreases with time, we can use the first piece of the general solution: thus

$$-(\gamma + \tfrac{1}{\tau}) = -29.606 \ \tfrac{1}{s}$$

Now, we know τ already, since

$$\tau = \frac{2m}{b} = \frac{2(0.98\,kg)}{29.20\ kg/s} = 0.0671\ s$$

And so

$$-\left(\gamma + \frac{1}{0.0671\,s}\right) = -29.606\ \frac{1}{s}$$

Solve for γ; because it is related to spring const k.

$$\gamma = 29.606\ \frac{1}{s} - \frac{1}{0.0671}\ \frac{1}{s}$$

$$= 29.606\ \frac{1}{s} - 14.898\ \frac{1}{s}$$

$$\gamma = 14.708\ \frac{1}{s}$$

Now

$$\gamma = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$$

Re-arrange to solve for k:

$$\gamma^2 = \frac{b^2}{4m^2} - \frac{k}{m}$$

$$\frac{k}{m} = \frac{b^2}{4m^2} - \gamma^2$$

$$k = \left(\frac{b^2}{4m^2} - \gamma^2\right) m$$

$$= \left[\frac{\left(29.20\ \frac{kg}{s}\right)^2}{4\,(0.98\,kg)^2} - \left(14.708\ \frac{1}{s}\right)^2\right](0.98\,kg)$$

$$\boxed{k = 5.51\ \frac{kg}{s^2} = 5.51\ \frac{N}{m}}$$