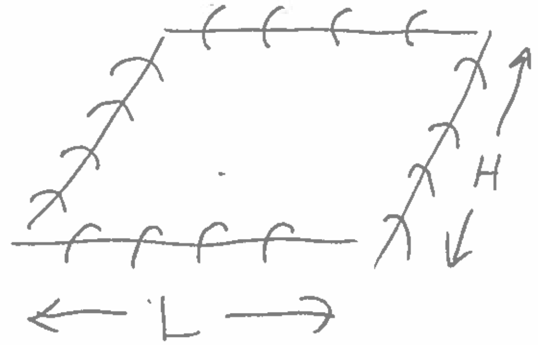

Helen makes a plate of thin metal

$L = 2.55$ m
$H = 1.00$ m
$M = 0.255$ kg

So $\sigma = \frac{M}{L \cdot H} = 0.100 \ \mathrm{kg/m^2}$

The plate is clamped around the edges so that they cannot vibrate or move. A wave crosses the plate from left to right in $\Delta t = 1.9922$ millisec. Thus the speed of waves in the plate is

$$V = \frac{L}{\Delta t} = \frac{2.55 \text{ m}}{1.9922 \times 10^{-3} \text{ s}} = 1280 \ \frac{\text{m}}{\text{s}}$$

We can now compute surface tension

$$V = \sqrt{\frac{S}{\sigma}} \Rightarrow S = \sigma V^2 = \left(0.100 \ \frac{\text{kg}}{\text{m}^2}\right)\left(1280 \ \frac{\text{m}}{\text{s}}\right)^2$$

$$= 1.638 \times 10^5 \ \frac{\text{N}}{\text{m}}$$

What are the lowest angular frequencies of normal modes of vibration in this plate?

- edges can't move $\Rightarrow z(x,y,t) = 0$ at $x = 0 \quad x = L$
$y = 0 \quad y = H$

$$\rightarrow \quad z = A \sin\left(\frac{n_x \pi}{L} x - \omega t\right) \sin\left(\frac{n_y \pi}{H} y - \omega t\right)$$

- use wave eqn

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{V^2} \frac{\partial^2 z}{\partial t^2} \quad \longrightarrow$$

$$-\frac{n_x^2\pi^2}{L^2}A\sin\left(\frac{n_x\pi}{L}x-\omega t\right)\sin\left(\frac{n_y\pi}{H}y-\omega t\right)-\frac{n_y^2\pi^2}{H^2}A\sin\left(\frac{\pi n_x}{L}-\omega t\right)\sin\left(\frac{\pi n_y}{H}-\omega t\right)$$

$$=\frac{1}{V^2}\left(-\omega^2 A\sin\left(\frac{n_x\pi}{L}x-\omega t\right)\sin\left(\frac{n_y\pi}{H}y-\omega t\right)\right)$$

which simplifies to

$$\frac{\pi^2 n_x^2}{L^2}+\frac{\pi^2 n_y^2}{H^2}=\frac{\omega^2}{V^2}$$

$$\Rightarrow \omega = V\sqrt{\frac{\pi^2 n_x^2}{L^2}+\frac{\pi^2 n_y^2}{H^2}}$$

So, for different choices of $n_x = 1,2,3,4,\ldots$ and $n_y = 1,2,3,4,\ldots$ we end up with a range of ω. To find the 8 lowest angular frequencies, just try a bunch and sort them.

in order	n_x	n_y	$\rightarrow$	ω (rad/s)
1	1	1		4319
2	2	1		5110
3	3	1		6208
4	4	1		7480
5	1	2		8195
6	2	2		8638
7	5	1		8850
8	3	2		9330