

Bernoulli Problem #1, page 19.

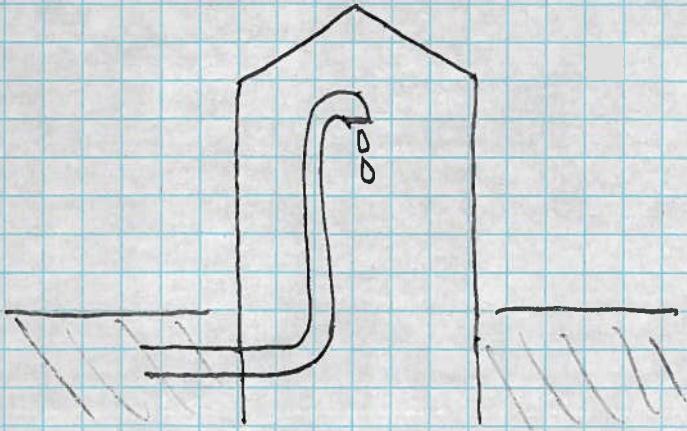
A pipe of radius 2.5 cm carries water through the basement of a two-story home at speed $0.90 \frac{\text{m}}{\text{s}}$ and gauge pressure 0.90 atm . Bathroom faucet on second floor, 8.2 m above basement, is turned on. What is speed of water coming out of faucet?

If you read problem and imagine this picture,

then the answer is very simple:

if all water flowing into one single pipe flows out at the top, then the continuous fluid in the pipe must obey

$$(\text{Area}_1)(v_1) = (\text{Area}_2)(v_2)$$



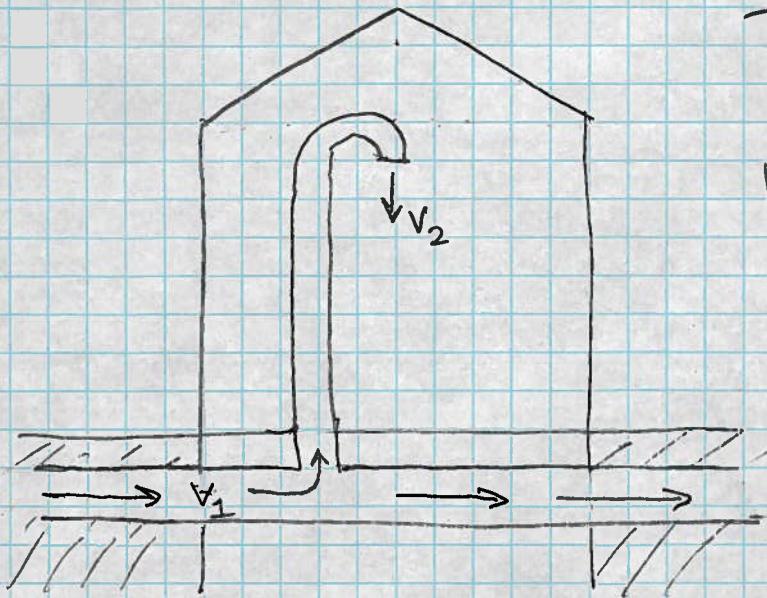
where these are Areas of cross section of the pipe. There is only one radius of pipe mentioned ... so must conclude pipe has constant size

→ constant area

→ constant velocity

In this case, water flows in at bottom $v_1 = 0.90 \frac{\text{m}}{\text{s}}$
out at top $v_2 = 0.90 \frac{\text{m}}{\text{s}}$

But my picture is the wrong one - that was my mistake!



The correct picture is shown here:
 a big pipe carrying water at $v_1 = 0.90 \frac{m}{s}$
 has a smaller connecting pipe attached;
some of the water from the big pipe
 enters the little pipe. In this case,
 the water splits - only some goes into
 little pipe. Since there is not a
 continuous, single flow, we don't
 know velocity of water in little pipe.

Here's what we do know.

$$\frac{1}{2} \rho v_1^2 + \rho g y_1 + P_1 = \frac{1}{2} \rho v_2^2 + \rho g y_2 + P_2$$

$$\rho = 1000 \frac{\text{kg}}{\text{m}^3} \quad \text{fresh water}$$

$$v_1 = 0.90 \frac{m}{s}$$

$$y_1 = 0 \text{ m} \quad \text{above basement}$$

$$y_2 = 8.2 \text{ m} \quad \text{above basement}$$

$$P_1 = 0.9 \text{ atm} \quad \text{gauge pressure} = \text{above air pressure}$$

$$P_2 = \text{air pressure, open to air}$$

$$\rightarrow P_1 - P_2 = 0.9 \text{ atm} = 9 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

We need to solve for v_2 , velocity at faucet on 2nd floor.

Rearrange

$$\frac{1}{2} \rho v_2^2 = \frac{1}{2} \rho v_1^2 + \rho g(y_1 - y_2) + (P_1 - P_2)$$



Now we can put in numbers

$$\frac{1}{2} \left(1000 \frac{\text{kg}}{\text{m}^3}\right) V_2^2 = \frac{1}{2} \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(0.90 \frac{\text{m}}{\text{s}}\right)^2$$

$$+ \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(9.8 \frac{\text{m}}{\text{s}^2}\right) (0\text{m} - 8.2\text{m})$$
$$+ (9 \times 10^4 \frac{\text{N}}{\text{m}^2})$$

$$\left(500 \frac{\text{kg}}{\text{m}^3}\right) V_2^2 = 405 \frac{\text{kg}}{\text{m} \cdot \text{s}^2} - 80,360 \frac{\text{kg}}{\text{m} \cdot \text{s}^2} + 9 \times 10^4 \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$$
$$= 10,045 \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$$

$$V_2 = \sqrt{\frac{10,045 \frac{\text{kg}}{\text{m} \cdot \text{s}^2}}{500 \frac{\text{kg}}{\text{m}^3}}}$$
$$= 4.48 \frac{\text{m}}{\text{s}}$$

but one input value, $P_1 = 0.9 \text{ atm}$, given to 1 significant figure, so proper form is

$$V_2 = 4 \frac{\text{m}}{\text{s}}$$