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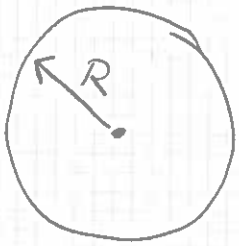
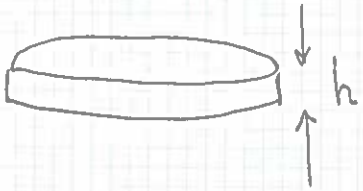
Is our currency being Debased?

Purpose: There are rumors that the gold content of the kingdom's coins has been decreasing at 5% per year for the past decade. We will measure coins from recent mintings to test this claim.

Measurements:





We measured three coins each produced in 5 recent mintings. Using a caliper, we measured the radius R and height h of each coin. All values in mm. We also measured the mass of each coin on a money-lender's balance, in units of grains.

Year	Coin 1 R	Coin 1 h	Coin 1 m	Coin 2 R	Coin 2 h	Coin 2 m	Coin 3 R	Coin 3 h	Coin 3 m	Average R	Average h	Average m
982	41	2.1	590	40	1.9	517	42	1.8	544			
984	39	2.2	546	38	2.2	531	37	2.1	474			
987	42	2.3	670	41	2.2	612	43	2.3	695			
989	40	2.1	543	43	2.0	583	42	1.9	515			
991	43	2.2	630	42	1.9	527	44	2.1	643			

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Year	average R (mm)	average h (mm)	average m (grain)	average ρ (grain/mm^3)
982	41.0 ± 1.0	1.93 ± 0.15	550 ± 37	0.054 ± 0.011
984	38.0 ± 1.0	2.17 ± 0.06	517 ± 38	0.053 ± 0.008
987	42.0 ± 1.0	2.27 ± 0.06	659 ± 43	0.052 ± 0.007
989	41.7 ± 1.5	2.00 ± 0.10	547 ± 34	0.050 ± 0.009
991	43.0 ± 1.0	2.07 ± 0.15	600 ± 63	0.050 ± 0.012

In order to compute density ρ, we use formula

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi R^2 h}$$

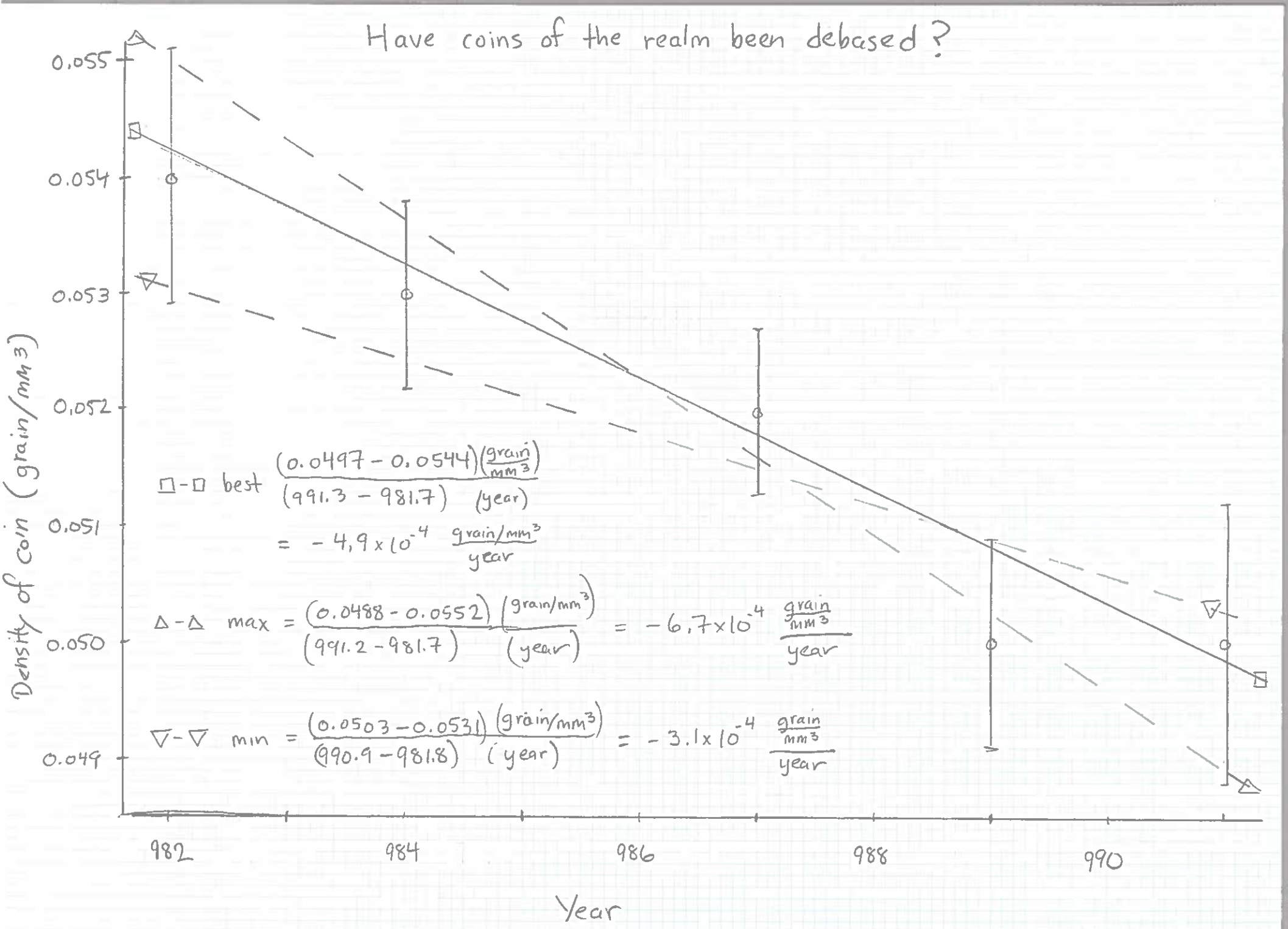
Therefore, the uncertainty in density is given by

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 2\frac{\Delta R}{R} + \frac{\Delta h}{h}$$

These average densities and uncertainties are written in table above.

Analysis: The claim is that the density of coins has dropped systematically as the amount of gold has decreased; the remainder being replaced by cheap, less dense metals, such as copper. In order to make a quantitative measurement of a systematic trend, we will graph the average density against time — the slope will express (change in density)/(per year).

Have coins of the realm been debased?



□-□ best $\frac{(0.0497 - 0.0544)\left(\frac{\text{grain}}{\text{mm}^3}\right)}{(991.3 - 981.7)\ (\text{year})}$

$= -4.9 \times 10^{-4}\ \frac{\text{grain/mm}^3}{\text{year}}$

△-△ max $= \frac{(0.0488 - 0.0552)\left(\frac{\text{grain/mm}^3}{\text{year}}\right)}{(991.2 - 981.7)} = -6.7 \times 10^{-4}\ \frac{\frac{\text{grain}}{\text{mm}^3}}{\text{year}}$

▽-▽ min $= \frac{(0.0503 - 0.0531)\left(\frac{\text{grain/mm}^3}{\text{year}}\right)}{(990.9 - 981.8)} = -3.1 \times 10^{-4}\ \frac{\frac{\text{grain}}{\text{mm}^3}}{\text{year}}$

Measuring the change in average properties of the coins over time (see graph), we find that the density is changing at

$$\text{best}: -4.9 \times 10^{-4} \ \frac{\text{grain/mm}^3}{\text{year}}$$

$$\text{min}: -3.1 \times 10^{-4} \ \frac{\text{grain/mm}^3}{\text{year}} \qquad \left|\frac{\text{max}-\text{min}}{2}\right| = 1.8 \times 10^{-4}$$

$$\text{max}: -6.7 \times 10^{-4} \ \frac{\text{grain/mm}^3}{\text{year}}$$

or

$$\text{rate of change} = \left(-4.9 \pm 1.8\right) \times 10^{-4} \ \frac{\text{grain/mm}^3}{\text{year}}$$

Conclusion: The density of the coins is decreasing with time, as the rate is not consistent with a value of 0.0. The percentage change per year is

$$\left[\frac{\left(-4.9 \pm 1.8\right) \times 10^{-4} \ \frac{\text{grain/mm}}{\text{year}}}{0.054 \ \text{grain/mm}}\right] * 100\% = (9.1 \pm 3.3)\%/\text{yr}$$

This rate is larger than the claim of a 5%/year change in gold content; even the minimum estimate of 5.8%/year is larger than 5%/year.

However, we have only measured a change in bulk density. In order to translate this into a change in gold content, we would need to know the densities of both gold and the base metals which are being mixed into the coins.