$$\begin{aligned} y_1(x,t) &= A_1\cos(k_1x \ - \ \omega_1 t \ + \ \phi_1) \\ y_2(x,t) &= A_2\cos(k_2x \ - \ \omega_2 t \ + \ \phi_2) \end{aligned}$$

$$y_1(x,t)+y_2(x,y) \quad = \quad \text{... messy ...}$$

$$\begin{aligned} y_1(x,t) &= A\cos(kx \ - \ \omega t \ + \ \phi_1) \\ y_2(x,t) &= A\cos(kx \ - \ \omega t \ + \ \phi_2) \end{aligned}$$

$$\Phi \quad \equiv \quad \phi_2 - \phi_1$$

$$\begin{aligned} y_1(x,t) &= A\cos(kx \ - \ \omega t) \\ y_2(x,t) &= A\cos(kx \ - \ \omega t \ + \ \Phi) \end{aligned}$$

$$\cos(\alpha) \ + \ \cos(\beta) \quad = \quad 2\cos\left(\frac{\alpha+\beta}{2}\right)\ \cos\left(\frac{\alpha-\beta}{2}\right)$$

$$y_1 \ + \ y_2 \quad = \quad 2\ \cos\left(\frac{\Phi}{2}\right)\ A\ \cos\left(kx \ - \ \omega t \ + \ \frac{\Phi}{2}\right)$$

$$\begin{aligned} y_1(x,t) &= A_1\cos(k_1x \ - \ \omega_1 t) \\ y_2(x,t) &= A_2\cos(k_2x \ - \ \omega_2 t) \end{aligned}$$

$$\omega_1 \quad \simeq \quad \omega_2$$

$$y_1 \ + \ y_2 \quad = \quad A\ \cos\left(\frac{\omega_1-\omega_2}{2}\,t\right)\ \cos\left(k_{\rm avg}x \ - \ \omega_{\rm avg}t\right)$$

$$\begin{aligned} y_1(x,t) &= A\cos(k_1x \ - \ \omega_1 t \ + \ \phi_1) \\ y_2(x,t) &= A\cos(k_2x \ - \ \omega_2 t \ + \ \phi_2) \end{aligned}$$

$$\begin{aligned} y_1(x,t) &= A\cos(\omega_1 t) \\ y_2(x,t) &= A\cos(\omega_2 t) \end{aligned}$$

$$y_1 \ + \ y_2 \quad = \quad 2A\ \cos\left(\frac{\omega_1-\omega_2}{2}\,t\right)\ \cos\left(\frac{\omega_1+\omega_2}{2}\,t\right)$$

$$y_1 \ + \ y_2 \quad = \quad 2A\ \cos\left(\frac{\omega_1-\omega_2}{2}\,t\right)\ \cos\left(\omega_{\rm avg}t\right)$$

$$\omega \quad = \quad 2\pi\,f$$

$$\begin{aligned} y_1(x,t) &= A\cos(2\pi f_1\,t) \\ y_2(x,t) &= A\cos(2\pi f_2\,t) \end{aligned}$$

$$y_1 \;+\; y_2 \quad = \quad 2A\,\cos\left(2\pi(1\ \mathrm{Hz})\,t\right)\,\cos\left(2\pi(499\ \mathrm{Hz})\,t\right)$$

$$f_{\mathrm{beat}} \quad = \quad 2\,\left(\frac{f_1 - f_2}{2}\right) \quad = \quad (f_1 \;-\; f_2)$$