$$
\begin{aligned}
y_1(x,t) &= A_1 \cos(k_1 x \ - \ \omega_1 t \ + \ \phi_1) \\
y_2(x,t) &= A_2 \cos(k_2 x \ - \ \omega_2 t \ + \ \phi_2)
\end{aligned}
$$

$$
y_1(x,t) + y_2(x,y) \quad = \quad \text{... messy ...}
$$

$$
\begin{aligned}
y_1(x,t) &= A \cos(kx \ - \ \omega t \ + \ \phi_1) \\
y_2(x,t) &= A \cos(kx \ - \ \omega t \ + \ \phi_2)
\end{aligned}
$$

$$
\Phi \quad \equiv \quad \phi_1 - \phi_2
$$

$$
\begin{aligned}
y_1(x,t) &= A \cos(kx \ - \ \omega t) \\
y_2(x,t) &= A \cos(kx \ - \ \omega t \ + \ \Phi)
\end{aligned}
$$

$$
\cos(\alpha) \ + \ \cos(\beta) \quad = \quad 2 \cos\left(\frac{\alpha+\beta}{2}\right) \cos\left(\frac{\alpha-\beta}{2}\right)
$$

$$
y_1 \ + \ y_2 \quad = \quad 2 \ \cos\left(\frac{\Phi}{2}\right) \ A \ \cos\left(kx \ - \ \omega t \ + \ \frac{\Phi}{2}\right)
$$

$$
\begin{aligned}
y_1(x,t) &= A_1 \cos(k_1 x \ - \ \omega_1 t) \\
y_2(x,t) &= A_2 \cos(k_2 x \ - \ \omega_2 t)
\end{aligned}
$$

$$
\omega_1 \quad \simeq \quad \omega_2
$$

$$
y_1 \ + \ y_2 \quad = \quad A \ \cos\left(\frac{\omega_1 - \omega_2}{2}\, t\right) \cos\left(k_{\text{avg}} x \ - \ \omega_{\text{avg}} t\right)
$$

$$
\begin{aligned}
y_1(x,t) &= A \cos(k_1 x \ - \ \omega_1 t \ + \ \phi_1) \\
y_2(x,t) &= A \cos(k_2 x \ - \ \omega_2 t \ + \ \phi_2)
\end{aligned}
$$

$$
\begin{aligned}
y_1(x,t) &= A \cos(\omega_1 t) \\
y_2(x,t) &= A \cos(\omega_2 t)
\end{aligned}
$$

$$
y_1 \ + \ y_2 \quad = \quad 2A \ \cos\left(\frac{\omega_1 - \omega_2}{2}\, t\right) \cos\left(\frac{\omega_1 + \omega_2}{2}\, t\right)
$$

$$
y_1 \ + \ y_2 \quad = \quad 2A \ \cos\left(\frac{\omega_1 - \omega_2}{2}\, t\right) \cos\left(\omega_{\text{avg}} t\right)
$$

$$
\omega \quad = \quad 2\pi f
$$

$$
\begin{aligned}
y_1(x,t) &= A \cos(2\pi f_1 \, t) \\
y_2(x,t) &= A \cos(2\pi f_2 \, t)
\end{aligned}
$$

$$y_1 \; + \; y_2 \quad = \quad 2A \cos\left(2\pi(1 \text{ Hz})\, t\right) \cos\left(2\pi(499 \text{ Hz})\, t\right)$$

$$f_{\text{beat}} \quad = \quad 2 \left(\frac{f_1 - f_2}{2} \right) \quad = \quad (f_1 \; - \; f_2)$$