$$\vec{F} = -k\vec{x}$$

$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x}$$

$$m\frac{d^2\vec{x}}{dt^2} + k\vec{x} = 0$$

$$\vec{x}(t) = \vec{A}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\vec{F}_{\text{resist}} = -b\vec{v}$$

$$\vec{F} = -k\vec{x} - b\vec{v}$$

$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x} - b\frac{d\vec{x}}{dt}$$

$$m\frac{d^2\vec{x}}{dt^2} + b\frac{d\vec{x}}{dt} + k\vec{x} = 0$$

$$x(t) = A\cos(\omega t + \phi)e^{-t/\tau} \qquad ???$$

$$\frac{dx}{dt} = -\frac{1}{\tau}A\cos(\omega t + \phi)e^{-t/\tau} - \omega A\sin(\omega t + \phi)e^{-t/\tau}$$

$$\begin{aligned}\frac{d^2x}{dt^2} = \quad & \frac{1}{\tau^2}A\cos(\omega t + \phi)e^{-t/\tau} \\ & + \frac{\omega}{\tau}A\sin(\omega t + \phi)e^{-t/\tau} \\ & + \frac{\omega}{\tau}A\sin(\omega t + \phi)e^{-t/\tau} \\ & - \omega^2 A\cos(\omega t + \phi)e^{-t/\tau}\end{aligned}$$

$$\frac{m}{\tau^2} - m\omega^2 - \frac{b}{\tau} + k = 0$$

$$\frac{2m\omega}{\tau} - b\omega = 0$$

$$\tau = \frac{2m}{b}$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$\frac{k}{m} - \frac{b^2}{4m^2} = 0$$

$$F = F_d \cos(\omega_d t) - kx - bv$$

$$m\frac{d^2x}{dt^2} - F_d \cos(\omega_d t) + b\frac{dx}{dt} + kx = 0$$

$$x(t) = A\cos(\omega_d t + \phi)$$

$$A = \frac{F_d}{m\sqrt{\left[\left(\omega_d^2 - \omega_0^2\right)^2 + \left(\frac{b\omega_d}{m}\right)^2\right]}}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$U(x) = 5x^2 \qquad \text{Joules}$$

$$U(x) = 3x^3 \qquad \text{Joules}$$

$$U(x) = 2x^4 \qquad \text{Joules}$$