Exam 1 Spring 2021 Solutions

Class Management I Instructor I Help

SP21 EXAM 1 -

Problem 1: A cart slides on a track in a straight line. The force acting on the cart is given by the formula $F(x) = \beta x^2$, where x is its distance from the starting point, in meters, and the force is in Newton (N).

$$[F] = [\beta][x^2]$$

$$N = [\beta]\, m^2 \Rightarrow [\beta] = \frac{N}{m^2}$$

What are the units of the constant β?

MultipleChoice :

1) N/m^2 ←
2) N/m
3) N·m
4) m^2/N
5) $N \cdot m^2$

Problem 2: A car is moving at a constant speed around a circular path.

Net force in direction of acceleration, towards center of circle.

What is the direction of the net force acting on the car?

MultipleChoice :

1) The net force on the car is zero since speed is constant
2) Away from the center of the circle
3) Tangent to the circle, opposite the direction of motion
4) Tangent to the circle, in the direction of motion
5) Towards the center of the circle ←

Problem 3: The mass and length of a thin cord was measured and linear mass density (mass per unit length) was computed.

1. Round uncertainty to one digit.
2. Make precision of result match uncertainty (same decimal place)

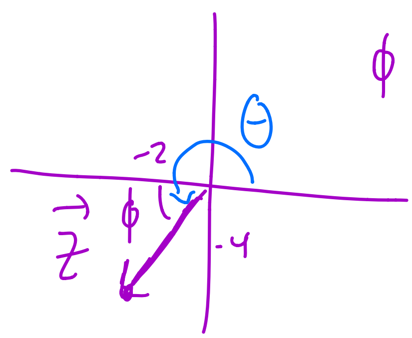
Which of the following measurements of linear mass density is expressed in the proper form, according to the labs done in class this term?
MultipleChoice :

1) (2.721 ± 0.221) kg/m
2) (2.72 ± 0.221) kg/m
3) (2.7 ± 0.02) kg/m
4) (2.72 ± 0.02) kg/m
5) (2.7 ± 0.22) kg/m

Problem 4: A student measures two vectors $\vec{U} = -4.00\hat{i} - 6.00\hat{j}$ and $\vec{W} = 2.00\hat{i} + 2.00\hat{j}$.

$\vec{Z} = -2\hat{i} - 4\hat{j}$

3RD Quadrant



$\phi = \tan^{-1}\left(\frac{-4}{-2}\right) = 63.4°$

$\theta = 180 + \phi$

What angle does the vector $\vec{Z} = \vec{U} + \vec{W}$ make with the +x axis?
MultipleChoice :

1) +243.4 degrees counterclockwise from +x
2) +63.4 degrees counterclockwise from +x
3) +153.4 degrees counterclockwise from +x
4) +206.6 degrees counterclockwise from +x
5) +26.6 degrees counterclockwise from +x

!

Problem 5: An object is tossed vertically upwards with an initial speed v_0.

given:

$V_{0y} = V_0$

$a = -g$

t

WANT!

V_{fy}

use eqn w/o Δy:

$V_{fy} = V_{0y} + at \Rightarrow \boxed{V_{fy} = V_0 - g t}$

Neglecting air resistance, what is the velocity of the object after being in the air for a time t? Take g to be the magnitude of the acceleration due to gravity, and take positive to be upwards.
MultipleChoice :

1) $v_0 + gt$
2) $-v_0 - gt$
3) $-v_0 + gt$
4) $v_0 t + \frac{1}{2}gt^2$
5) $v_0 - gt$
6) $v_0 t - \frac{1}{2}gt^2$

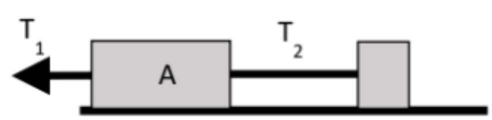
Problem 6: An object experiences a potential energy of the form $U(x) = -Ax + Bx^2$.

$$F_x = -\frac{dU}{dx} = -(-A + 2Bx)$$
$$= +A - 2Bx$$

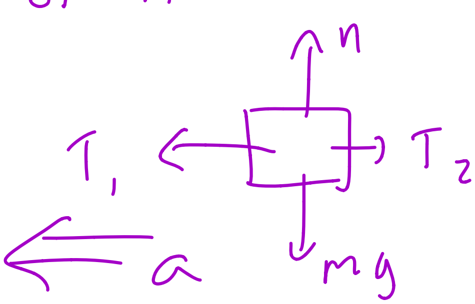
What is the conservative force corresponding to this potential energy?
MultipleChoice :

1) $-\frac{Ax^2}{2} + \frac{Bx^3}{3}$
2) $A - 2Bx$
3) $\frac{Ax^2}{2} - \frac{Bx^3}{3}$
4) $-A + 2Bx$
5) $-A + \frac{Bx}{2}$

Problem 7: Two boxes are connected to each other by a string. Box A is pulled to the left by another string, resulting in both boxes accelerating towards the left.



FBD of A:



Accelerating to left so net force towards left, so $T_1 > T_2$

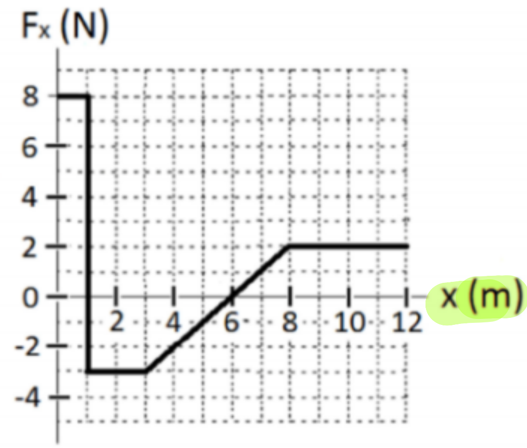
How does the magnitude of the tension T_1 compare to the magnitude of the tension T_2?
MultipleChoice :

1) $T_1 > T_2$
2) Whether T_1 is bigger or smaller than T_2 depends on the relative masses of the two blocks.
3) Whether T_1 is bigger or smaller than T_2 depends on whether there is kinetic friction between the surface and the blocks.
4) $T_1 = T_2$
5) $T_1 < T_2$

Problem 8: An object moving in the positive x direction is acted on by a net force F_x that changes with position as shown.

$\int F_x dx = W = \Delta K$

integrals are areas

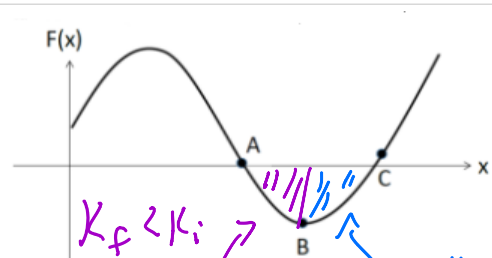


Which one statement is true?
MultipleChoice :

1) The slope of this graph corresponds to the acceleration of the object.
2) The slope of this graph corresponds to the change in velocity.
3) None of these statements is true.
4) The work done by this force can be determined by taking the slope of this graph.
5) The area under this graph corresponds to the change in velocity.
6) The area under this graph corresponds to the change in kinetic energy.

Problem 9: Consider the graph of the net force acting on an object as a function of position. This depicts the net force acting on a crate of mass m that starts from rest at the origin and moves along the positive x direction under the influence of this net force.

Recall Area = ΔK. Negative area is slowing down.



$K_f < K_i$ slowing down

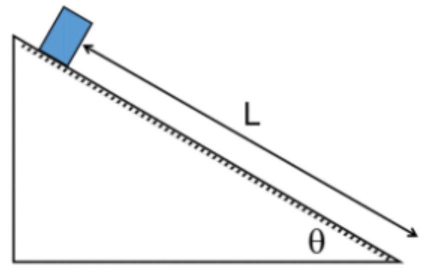
still slowing down

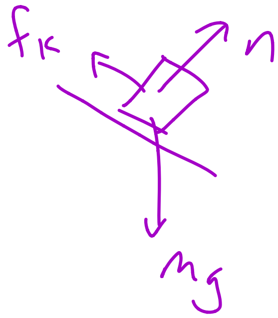
Rank the kinetic energy (K_A, K_B, and K_C) at the three points (A, B, and C) shown.
MultipleChoice :

1) $K_A > K_C > K_B$
2) $K_B > K_C > K_A$
3) $K_C > K_A > K_B$

4) $K_B > K_A > K_C$
5) $K_A > K_B > K_C$
6) $K_C > K_B > K_A$

Problem 10: A container with mass m is originally at rest on a ramp. The ramp makes an angle θ with the horizontal and the coefficient of kinetic friction between the container and the ramp is μ_k. After being released from rest, the container slides a distance L along the ramp.





$f_K = \mu_K n$

$n_{ramp} = mg\cos\theta \Rightarrow f_K = \mu_K mg\cos\theta$

What is the magnitude of the friction force acting on the container as it slides down the ramp?
MultipleChoice :

1) The friction force will depend on L
2) 0
3) $\mu_k mg \sin\theta$
4) $\mu_k mg \cos\theta$
5) $mg \sin\theta$
6) $mg \cos\theta$

Problem 11: A force $\vec{F} = \left(2.0\hat{i} - 3.0\hat{j}\right)$ N acts on an object of mass 3.0 kg as it undergoes a displacement of $\vec{d} = \left(-4.0\hat{i} + 3.0\hat{j}\right)$ m.

$W = \vec{F} \cdot \vec{d} = (-8 - 9)\,J = -17\,J$

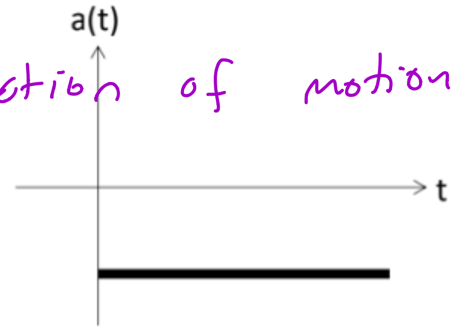
dot product

NO $\hat{i}$ OR $\hat{j}$ IN DOT PRODUCT Result!!!
$\Rightarrow$ SCALAR!

What is the work done by this force during this motion?
MultipleChoice :

1) $\left(-8.0\hat{i} - 9.0\hat{j}\right)$ J
2) $+1.0$ J
3) $\left(-2.0\hat{i} - 0.0\hat{j}\right)$ J
4) -17.0 J
5) -2.0 J
6) You need the initial speed to determine the work done by this force.

Problem 12: Consider the acceleration versus time graph shown for a car confined to move in one dimension. Say that the car was initially moving with a velocity of -3.0 m/s at $t = 0.0$ seconds.



- $V_0 = -3\frac{m}{s}$ ← negative, initial direction of motion.
- $\vec{a}$ also negative. So object will speed up.

Which statement best describes the motion of the car for the time shown in the diagram?
MultipleChoice :

1) The car heads in the negative direction and speeds up the entire time.
2) The car heads in the positive direction and speeds up the entire time.
3) The car initially heads in the negative direction, then it slows down and reverses direction of motion, eventually heading in the positive direction and speeding up.
4) The car heads in the positive direction and slows down the entire time.
5) The car initially heads in the positive direction, then it slows down and reverses direction of motion, eventually heading in the negative direction and speeding up.
6) The car heads in the negative direction and slows down the entire time.

Problem 13: A block of mass m slides across a level surface with coefficient of kinetic friction μ_k. It travels a distance d through the rough patch before coming to rest.

CoE : $\frac{1}{2}mv_0^2 = \underbrace{\mu_k m g d}_{(\Delta U_{int})}$

$\therefore \; v_0 = \sqrt{2\mu_k g d}$

What was the initial speed of the block?
MultipleChoice :

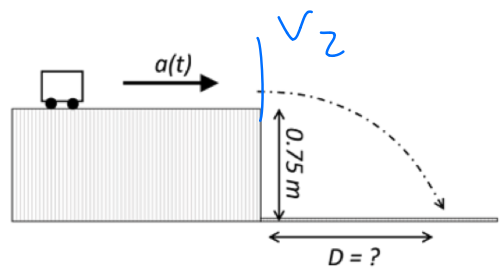
1) $\dfrac{d^2}{2\mu_k g}$
2) $\sqrt{\mu_k g d}$
3) $\sqrt{2\mu_k m g d}$
4) $\sqrt{2\mu_k g d}$
5) $2\sqrt{\mu_k g d}\cos(\theta)$

Problem 14: Long Problem 1

A toy cart is moving in one dimension along a level tabletop with a **non-constant** acceleration given by $a(t) = At^2\hat{i}$ where the constant $A = 3.45$ m/s^4, and t is measured in seconds.

The cart has an initial velocity of +(2.40 m/s)$\hat{i}$ at $t = 0.00$ seconds. It gets to the edge of the tabletop at $t = 0.50$ seconds.

Upon reaching the edge of the tabletop, it is launched horizontally off and lands on the level ground below, as demonstrated in the illustration. While in the air, air resistance can be ignored, and the only acceleration is due to gravity.



The height of the tabletop is 0.750 meters. How far from the base of the tabletop does the cart land? Express your numerical answer to three significant digits, and include units in your answer.

1D MOTION CALCULUS: Get v(0.5) (6 PTS)

$a(t) = At^2$ $\quad v_i = 2.40 \text{ m/s}^2$, $A = 3.45$

$$V_2 \equiv v(t) = \int a\,dt = \int At^2\,dt = \left.\frac{At^3}{3}\right|_0^{0.5} + V_i$$

$$V_2 = 2.54 \frac{m}{s}$$

2D PROJECTILE MOTION HORIZONTAL LAUNCH

$\hat{x}$ (6 PTS)

$\Delta x = V_x t_{air}$

$D = V_2 t_{air}$

get t FROM $\hat{y}$

$\hat{y}$ (6 PTS)

$a = -9.8$

$y_0 = 0.75$

$y_f = 0$

$V_{y0} = 0$

$y_f = y_0 + V_{y0}t + \frac{1}{2}at_{air}^2$

$0 = 0.75 + 0 + \frac{1}{2}(-9.8)t_{air}^2$

$t_{air} = 0.3912$ sec

$\boxed{D = 0.995 \text{ m}}$

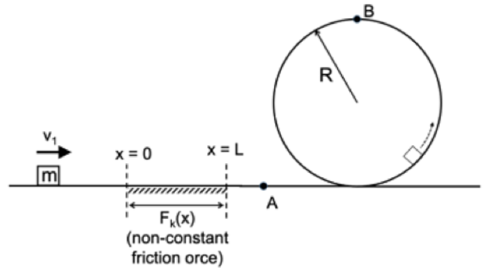
UNITS 1PT

MATH 1PT

Problem 15: Long Problem 2

Consider the situation depicted in the image. A crate of mass m is originally traveling with a speed v_1 along a level pathway. Then, it travels over a level friction patch of length L. The force of kinetic friction in the patch **varies with position** and is given by $F_k(x) = Cx^3$ where C is a positive constant, and $x = 0$ at the start of the friction patch.

After sliding through the friction patch, the crate travels around a frictionless loop-the-loop of radius R.



(a) What is speed of the crate when it reaches point A, once it gets through the friction patch and before going through the loop? Put your answer in terms of only the following: C, m, L, and v_1.

(b) Take the speed at point A to be v_A. What is the normal force on the crate when it is upside down at the top of the loop, at point B in the diagram? Put your answer in terms of only v_A, m, g, and R.

a) WORK - KE IN FRICTION PATCH 8 PTS

$$W = \int F_x \, dx = \Delta K \qquad : \qquad W = -\int_0^L Cx^3 \, dx = \Delta K$$

because f_k opposite dx

$$-\frac{CL^4}{4} = \frac{1}{2} m v_A^2 - \frac{1}{2} m v_1^2$$

$$\boxed{v_A = \sqrt{m v_1^2 - \frac{CL^4}{2}}}$$

b) CoE : N2L for vertical circle 12 PTS

CoE :

$$K_2 = K_{TOP} + U_{gTOP}$$

$$\frac{1}{2} m v_A^2 = \frac{1}{2} m v_{TOP}^2 + mg(2R)$$

$$v_{TOP}^2 = v_A^2 - 4gR$$

N2L @ TOP

n mg $a = \frac{v^2}{r}$ $+\downarrow$

$$n + mg = \frac{m v_{TOP}^2}{R}$$

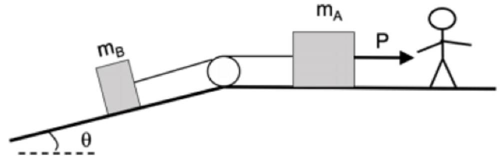
$$n = \frac{m v_{TOP}^2}{R} - mg$$

$$\boxed{n = \frac{m(v_A^2 - 4gR)}{R} - mg}$$

1

Problem 16: Long Problem 3

Two masses are joined by an ideal rope as shown in the diagram. One box of mass m_B is on a ramp that makes an angle θ with the horizontal. The other box has a mass m_A and is on a level surface. A physicist pulls on m_A with a horizontal force of magnitude P, causing both masses to accelerate together such that box A travels towards the right. All surfaces are frictionless.

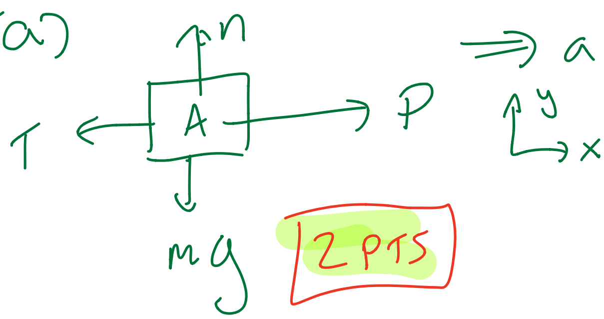


(a) Draw a clear free body diagram for each box.

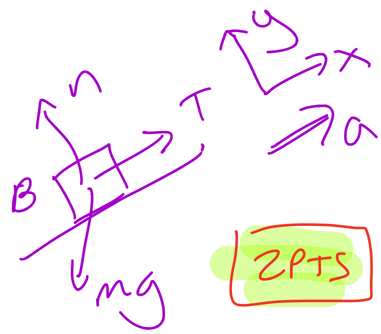
(b) Determine the magnitude of the tension in the rope between the two masses and the magnitude of the acceleration of the masses. Show all work. Put each answer **only** in terms of P, θ, m_A, m_B, and g.

(c) What is the normal force on box B?

(a)



2 PTS



2 PTS

(b) 12 PTS TOTAL

$$P - T = M_A a \qquad T - M_B g \sin\theta = M_B a$$

SOLVE: $T = (P - M_A a)$:

$$(P - M_A a) - M_B g \sin\theta = M_B a$$

$$\boxed{a = \frac{P - M_B g \sin\theta}{M_A + M_B}}$$

$\boxed{T = P - M_A a, \text{ where } a =}$

(c) $n - M_B g \cos\theta = 0$

$$\boxed{n = M_B g \cos\theta}$$

4 PTS