Answer Key UP1Sample Test #1 Fall 2017

Multiple choice

1. D

2. C

3. E

4. A

5. D

6. B

7. C

8. A

9. D

10. D

11. A

12. B

13. D

GOOD LUCK ON THE TEST!!!!

Long Problem example #1

14) 20 pts UFO

UFO moves in 1d with $v(t) = 4.00 - 9.00t^2$
for $t \geq 0$ (Assume $+\hat{i}$)

a) SI UNITS OF 4.00 & 9.00 terms

are: $4.00\,m/s$ m/s ✓

$9.00\,m/s^3$ so $(m/s^3)(s^2) = m/s$ ✓

b) UFO @ REST MOMENTARILY WHEN

$v(t) = 4\,m/s - (9\,m/s^3)t^2$

@ rest $v(t') = 0$

$0 = 4\,m/s - (9\,m/s^3)t^2$

$4\,m/s = (9\,m/s^3)t^2$

$t^2 = 4/9\,s^2$ $\Rightarrow \boxed{t = 2/3\,s}$

Ans

or $t = 0.667s$ @ 3 sig figs

14-1

c) Find $\vec{a}(3)$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\therefore a = -2(9\,m/s^3)t\,\hat{i} = (-18\,m/s^3)t\,\hat{i}$$

$$\vec{a}(3) = (-18\,m/s^3)(3\,s)\,\hat{i}$$

$$\boxed{\vec{a}(3) = -54\,m/s^2\,\hat{i}}$$

d) @ $t_0 = 0$ UFO @ $\vec{x}(0) = -7.00\,m\,\hat{i}$

Find $\vec{x}(t)$

$$\Delta x = \int_{t_0}^{t} v(t)\,dt = \int_0^t \left(4\,m/s - (9\,m/s^3)t'^2\right)dt'\,\hat{i}$$

$$\vec{x}(t) - \vec{x}(0) = \left[(4\,m/s)t' - (9\,m/s^3)\frac{t'^3}{3}\right]_0^t \hat{i}$$

$$= \left((4\,m/s)t - (3\,m/s^3)t^3\right)\hat{i} - 0$$

$$\therefore \vec{x}(t) = \left(\overset{-7m}{\cancel{\vec{x}(0)}} + (4\,m/s)t - (3\,m/s^3)t^3\right)\hat{i}$$

$$= \left(-7\,m + (4\,m/s)t - (3\,m/s^3)t^3\right)\hat{i}$$

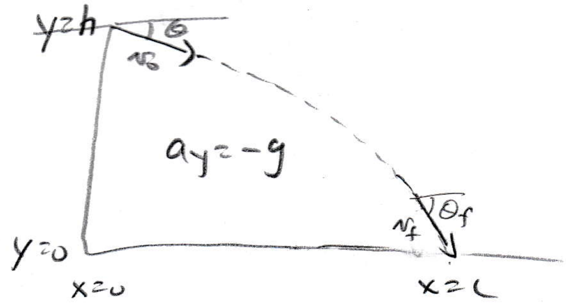
$$\boxed{\vec{x}(t) = (-7.00 + 4.00t - 3.00t^3)\,\hat{i}\ m}$$

@ 3 sig figs

Long Problem   example #2

15) Chucking stones off building

UNDERSTAND



Know

$v_0 = 90.0\ m/s$

$\theta = 35.0°$ below +x axis

Find

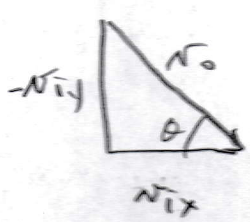
a) L

b) v_f, θ_f

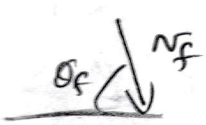
$\Delta y = 0 - h \qquad \Delta x = L - 0$

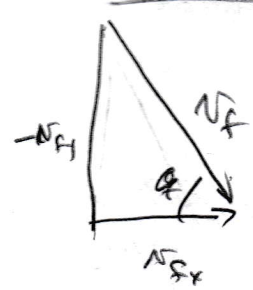
a) Sketch trajectory ⇒ see above **ANS**

@ launch



@ landing





$v_{ix} = v_{fx}$

since

$a_x = 0$

$$\boxed{\begin{aligned} v_{iy} &= -v_0 \sin\theta = -14.34\ m/s \\ v_{ix} &= v_0 \cos\theta = 20.45\ m/s \end{aligned}}$$

$v_{fy} = -v_f \sin\theta_f$

$v_{fx} = v_f \cos\theta_f$

NOTE Don't know / need time in air!

PLAN

b) Find range L by eliminating Δt

from $\Delta x = v_{ix} \Delta t$ and $\Delta y = v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$

c) Find θ_f using $\tan \theta_f = v_{fy}/v_{fx}$ AND

v_f using $v_{fy}^2 - v_{iy}^2 = 2 a_y \Delta y$ and

$v_f = \sqrt{v_{fy}^2 + v_{fx}^2}$ with $v_{fx} = v_{ix}$

Do

b) $\overset{0-h}{\cancel{\Delta y}} = v_{iy} \left(\frac{L}{v_{ix}}\right) - \frac{g}{2} \left(\frac{L}{v_{ix}}\right)^2$

$0 = h + \left(\frac{v_{iy}}{v_{ix}}\right) L - \frac{g}{2} \left(\frac{1}{v_{ix}}\right)^2 L^2$ $\qquad 0 = C + BL + AL^2$

solve via quadratic formula $\quad L = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$C = h \quad B = \frac{v_{iy}}{v_{ix}} \quad A = \frac{-g}{2} \frac{1}{v_{ix}^2}$

$L = \begin{cases} -123\text{ m} \\ +62.8\text{ m} \end{cases}$ $\quad$ ← − root, unphysical soln / ← + root, the answer

$\boxed{L = 62.8\text{ m}}$

$\boxed{\text{DO}}$

c) $v_{fy}^2 = v_{iy}^2 + 2\overset{-g}{a_y}\,\overset{0-h}{\Delta y}$

$v_{fy} = \sqrt{v_{iy}^2 + 2gh}$

$= \pm\sqrt{(-14.34)^2 + 2(9.81\,m/s^2)(90\,m)}$

$v_{fy} = -44.40\ m/s$ (- root since down)

$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$

$= \sqrt{(20.45\,m/s)^2 + (-44.40\,m/s)^2}$

$\boxed{v_f = 48.89\ m/s}$ is speed @ impact

Ans

$\theta_f = \tan^{-1}\left(-\dfrac{44.40\,m/s}{20.45\,m/s}\right) = -65.27°$

$\boxed{\theta_f = 65.3° \text{ below } +x \text{ axis}}$

Ans

Check:

$v_0 \cos\theta = v_f \cos\theta_f$ $\quad 20.45\,m/s = 20.45\,m/s$ ✓

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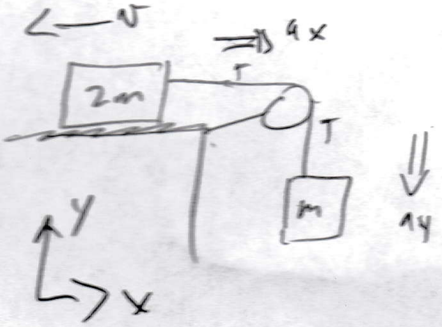
16) 20 pts

Long Problem example #3

Modified Atwood. w friction



[UNDERSTAND]

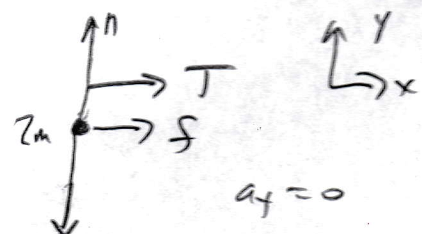
N_i T's ←

know μ_k, m, 2m

Find a & T in initial state.

motion: slow down & stop 2m.

FBD 2m:

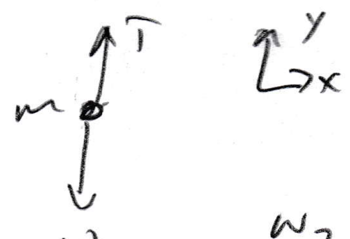


$a_y = 0$

$f = \mu n$

$w_1 = 2mg$

FBD m:



$w_2 = mg$

$\Sigma F_x = T + f$

$\Sigma F_y = n - 2mg$

$\Sigma F_x = 0$

$\Sigma F_y = T - mg$

CONSTRAINT: (objects accelerate together)

$a_x = +a \qquad a_y = -a$

[PLAN]

i) Apply N2L in x & y to each block
ii) Eliminate T & solve for a
iii) sub in a) for hanging block to get T

DO

N2L for 2m:

i)

x: $T + f = (2m)a_x$

$T + \mu_k n = 2m(+a)$

$T = 2ma - \mu n$ (A)

y: $n - 2mg = 0$

$n = 2mg$ (B)

N2L for m

x: $0 = 0$

y: $T - mg = m(-a)$

$T = m(g - a)$ (C)

ii) COMBINE (A), (B) & (C) To eliminate T

$2\cancel{m}a - \mu(2\cancel{m}g) = \cancel{m}(g - a)$

$2a + a = g + 2\mu g$

$3a = g(1 + 2\mu)$

$\boxed{a = \frac{1+2\mu}{3} g}$ Ans

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iii)

$$T = m\left(g - \left(\frac{1+2\mu}{3}g\right)\right)$$

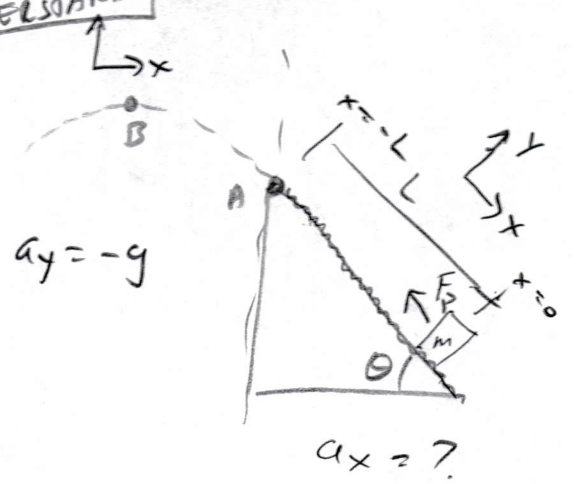
$$T = mg\left(\frac{3-1-2\mu}{3}\right)$$

$$\boxed{T = 2mg\left(\frac{1-\mu}{3}\right)}$$

17) Long problem example #4

20 pts   Accelerate off ramp

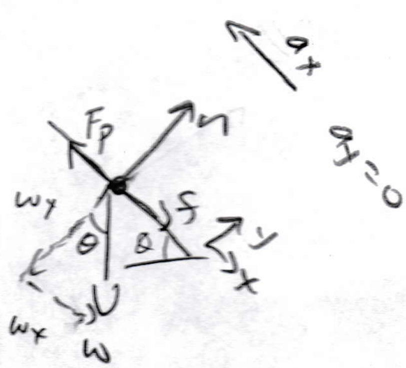
UNDERSTAND



know
$m = 2.00\,kg$
$L = 0.500\,m$
$F_p = 30.0\,N$
$\theta = 40.0°$
$\mu = 0.250$
$v_0 = 0\,m/s$

FIND time to reach B.
t_B

FBD m on ramp



$\Sigma F_x = -F_p + f + w_x$

$\Sigma F_y = n - w_y$

$f = \mu n$

$w = mg.$

$w_y = -w\cos\theta$
$w_x = w\sin\theta$

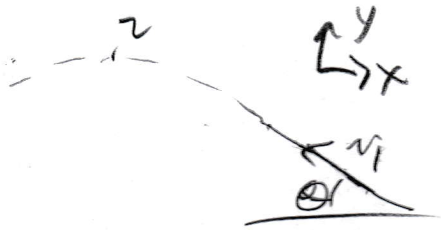
Kinematics on ramp
- don't know time or need it.

$v_{2x}^2 = v_{1x}^2 + 2a_x \Delta x$

$\Delta x = -L - 0$
$= -L$

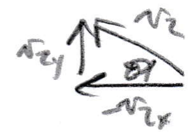
a_x comes from N2L

Projectile in air



$v_{1x} = -v_1 \cos\theta$

$v_{1y} = v_1 \sin\theta$



@ max height $v_y = 0$

$$\overset{0}{\cancel{v_{2y}}} = v_{1y} + \overset{-g}{\cancel{a_y}}\, t_2$$

$\boxed{\text{PLAN}}$

i) Find a_x on incline due to F_P & f
solving N2L in x & y

ii) use $v_f^2 = v_i^2 + 2a_x \cdot \Delta x$ to find launch speed

iii) launch velocity is launch speed combined with incline angle.

iv) @ max height $v_y = 0$
use y component of projectile motion to solve for t in $v_{f_y} = v_{i_y} + a_y t$

$\boxed{\text{Do}}$

i) $-F_p + f + w_x = m(-a_x)$ for $\Sigma F_x = m a_x$

$-F_p + \mu n + \overset{mg}{\cancel{w}} \sin\theta = m(-a_x)$

$n - w_y = m\cancel{(a_y)}^{\to 0}$

$n - w\cos\theta = 0$

$n = \overset{mg}{\cancel{w}}\cos\theta$

$-F_p + \mu mg\cos\theta + mg\sin\theta = -ma_x$

$\therefore a_x = +F_p/m - g(\mu\cos\theta + \sin\theta)$

For #'s given

$a_x = +\frac{30.0\,N}{2.00\,kg} - (9.80\,m/s^2)(0.250\cos 40 + \sin 40)$

$= +15.0\,m/s^2 - 8.18\,m/s^2$

$\boxed{a_x = 6.82\,m/s^2}$

$-x$ is up ramp ←

$\vec{a}_x = -a_x = -6.82\,m/s^2\,\hat{\imath}$

17-3

ii) $v_1^{\,2} = \cancel{v_0^{\,2}}^{\,0} + 2(-a_x)\cancel{\Delta x}^{\,-L-0}$

$v_1 = \sqrt{2(-a_x)(-L)} = \sqrt{2a_xL}$

$= \sqrt{2(6.82\,m/s^2)(0.500\,m)}$

$$\boxed{v_1 = 2.61\,m/s}$$

iii)

$v_{1x} = -v_1 \cos\theta$

$v_{1y} = v_1 \sin\theta$

@ max height $v_{2y} = 0$

$\cancel{v_{2y}}^{\,0} = v_{1y} - g t_2$

$\therefore t_2 = v_{1y}/g$

$$\boxed{t_2 = \frac{v_1 \sin\theta}{g}}$$ Ans

$$\boxed{t_2 = 0.171\,s}$$ Ans